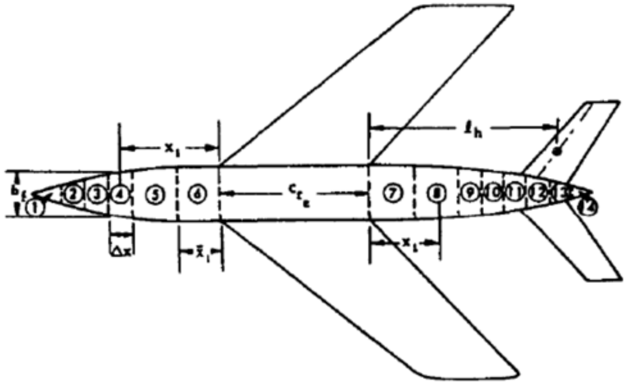
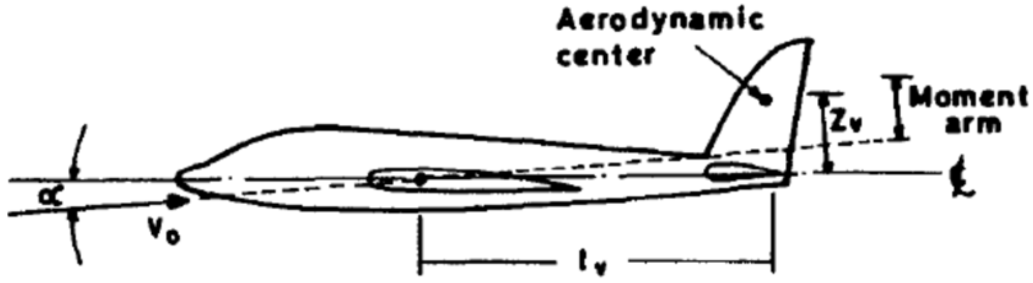


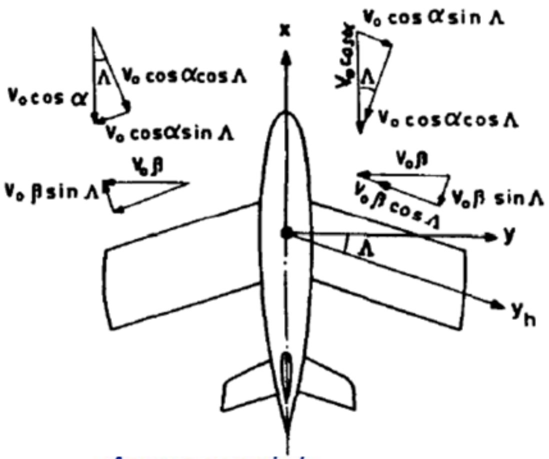


a) Low angle of attack

Estabilidad y Control Detallado
Derivadas Estabilidad Lateral-Direccional

Tema 14.3

Sergio Esteban Roncero
Departamento de Ingeniería Aeroespacial
Y Mecánica de Fluidos



Derivadas $C_{Y_\beta}, C_{L_\beta}, C_{N_\beta}, C_{Y_{T_\beta}}, C_{N_{T_\beta}}$

Sideslip Derivatives

Estimación Derivadas

- Contribución C_{Y_β}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje
- Contribución C_{L_β},
  - Ala: flecha, diedro
  - Vertical
  - Canard/horizontal/V-tail
  - Fuselaje
- Contribución C_{N_β}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje
- Contribución $C_{Y_{T_\beta}}$
- Contribución $C_{N_{T_\beta}}$

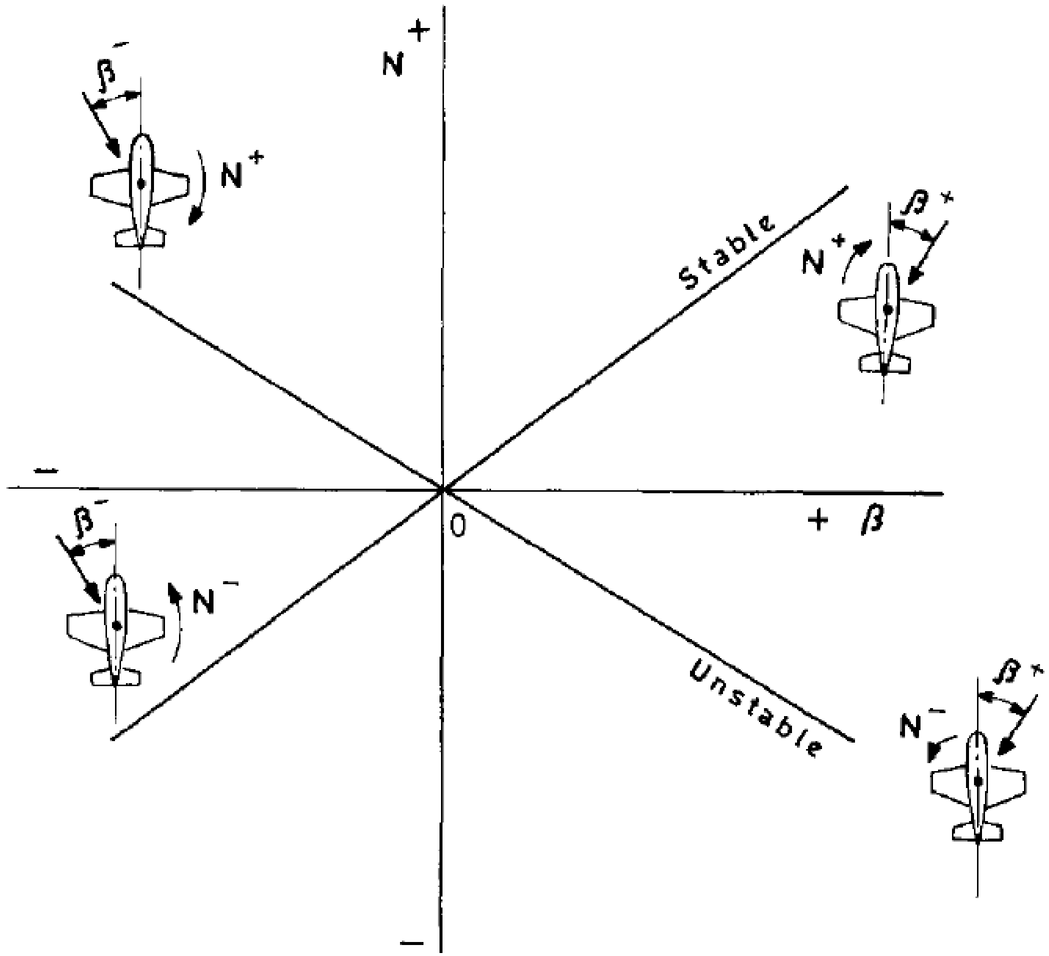
Derivadas en 1/rad si no se indica lo contrario

Si las derivadas no están en 1/rad hay que convertirlas

Estabilidad Direccional

Criterio Estabilidad Direccional

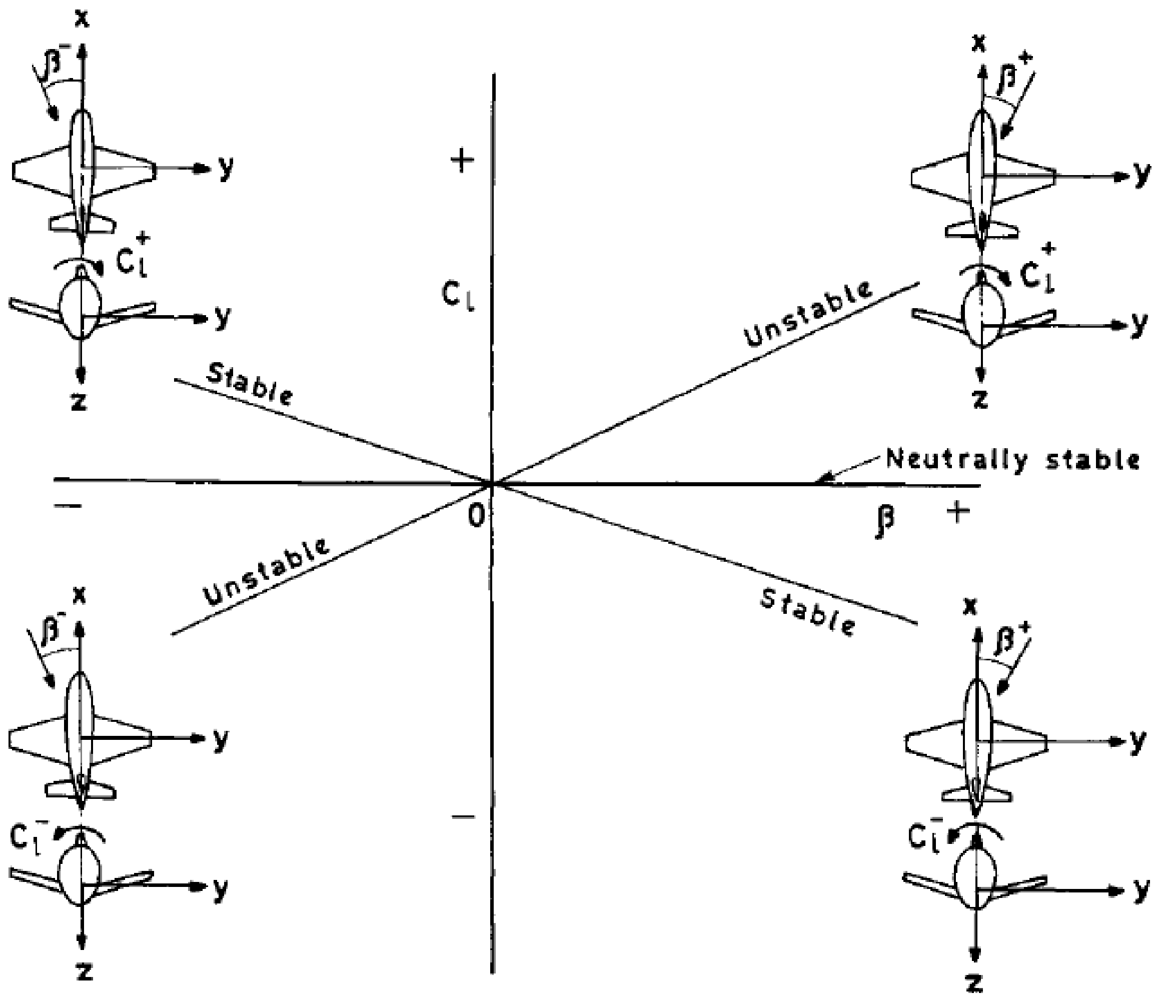
$$C_n = \frac{N}{qSb} \qquad C_{n\beta} = \frac{\partial C_n}{\partial \beta} \qquad C_{n\beta} > 0 \qquad N_\beta > 0$$



Estabilidad Lateral

Criterio Estabilidad Lateral

$$C_l = \frac{L}{qSb} \qquad C_{l\beta} = \frac{\partial C_l}{\partial \beta} \qquad C_{l\beta} < 0 \qquad L_\beta < 0$$



Lateral-Directional C_{Y_β}

Método I

Contribución total

$$C_{y_\beta} = \underbrace{C_{y_{\beta_W}}}_{\text{ala}} + \underbrace{C_{y_{\beta_f}}}_{\text{fuselaje}} + \underbrace{C_{y_{\beta_v}}}_{\text{vertical}}$$

Wing contribution

$$(C_{y_\beta})_{wing} = C_L^2 \frac{6 \tan\Lambda \sin\Lambda}{\pi\, AR(AR + 4\cos\Lambda)} \cdot \text{(per radian)}$$

AR – alargamiento ala
Λ – flecha ala

aproximación $\Rightarrow$ $C_{y_{\beta_W}} = -0.00573\left|\Gamma_W\right|$ $\Rightarrow$ 1/rad

Ref: Smetana

Lateral-Directional C_{Y_β}

Método I

fuselage contribution

$$(C_{Y\beta})_{fus} = -K_i (C_{L_\alpha})_{fus} \left(\frac{\text{Body Reference Area}}{S_W}\right)$$

$$C_{L_{\alpha Body}} = \frac{2(k_2 - k_1)S_o}{V_b^{2/3}}$$

Body Reference Area $=$ (fuselage volume)$^{2/3}$

K_i - wing-fuselage interference factor

Ref: Smetana

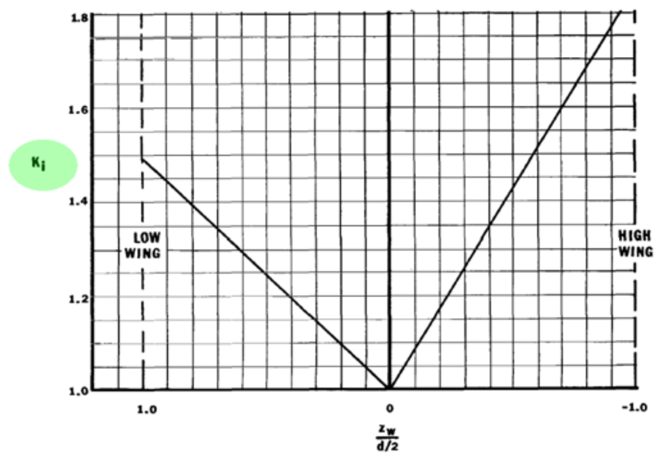


Fig B6

Fig A5

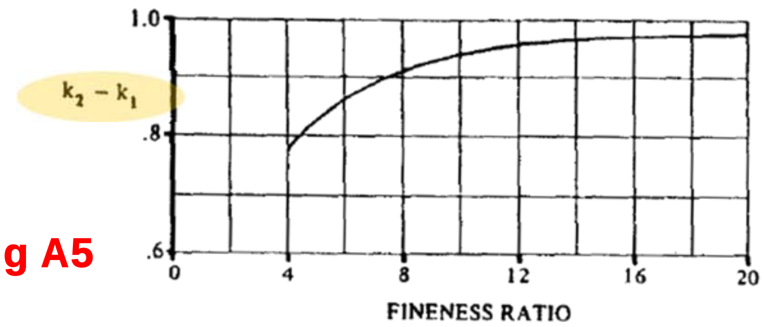


Fig. 3.6 Fuselage apparent mass coefficient.[1]

$C_{L\alpha,f}$

Ref: Smetana

$$C_{L\alpha_{Body}} = \frac{2(k_2 - k_1)S_o}{V_b^{2/3}}$$

$k_2 - k_1$ = apparent mass factor which is a function of fineness ratio (length/maximum thickness)

V_b = total body volume

S_o = cross sectional area at x_o

S_x = body cross sectional area at any body station

ℓ_b = body length.

x_1 = the body station where the parameter dS_x/dx first reaches its minimum value.

x_o = body station where flow ceases to be potential

Fig A5

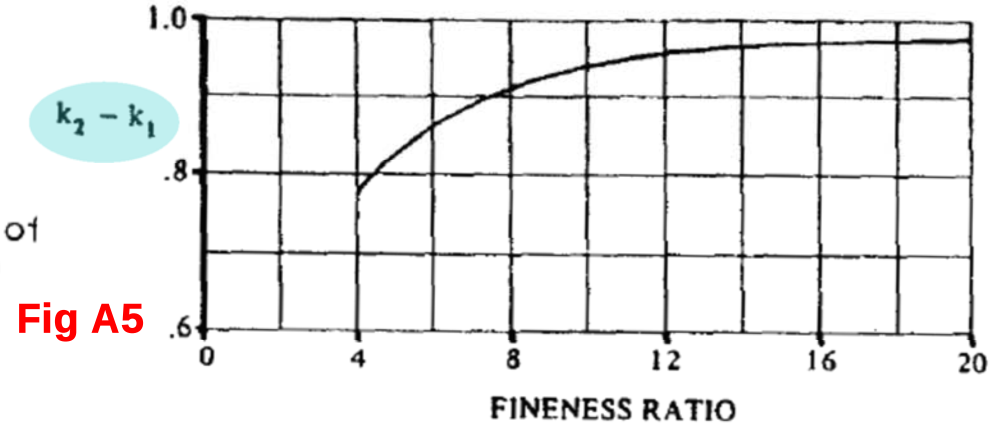


Fig. 3.6 Fuselage apparent mass coefficient.[1]

El $C_{L\alpha}$ del fuselaje se obtiene determinando el punto del fuselaje donde el flujo deja de ser potencial (X_0) el cual se determina en función de X_1 que es el punto del fuselaje donde la variación del fuselaje alcanza un mínimo en la variación de área, es decir cuando el fuselaje deja de aumentar

Fig A6

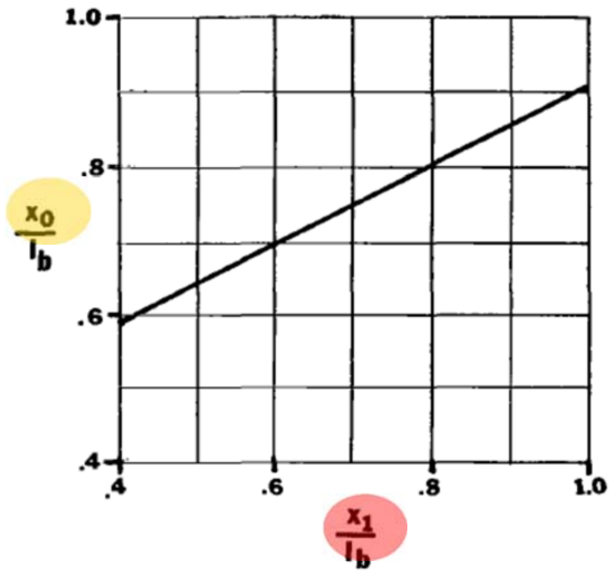
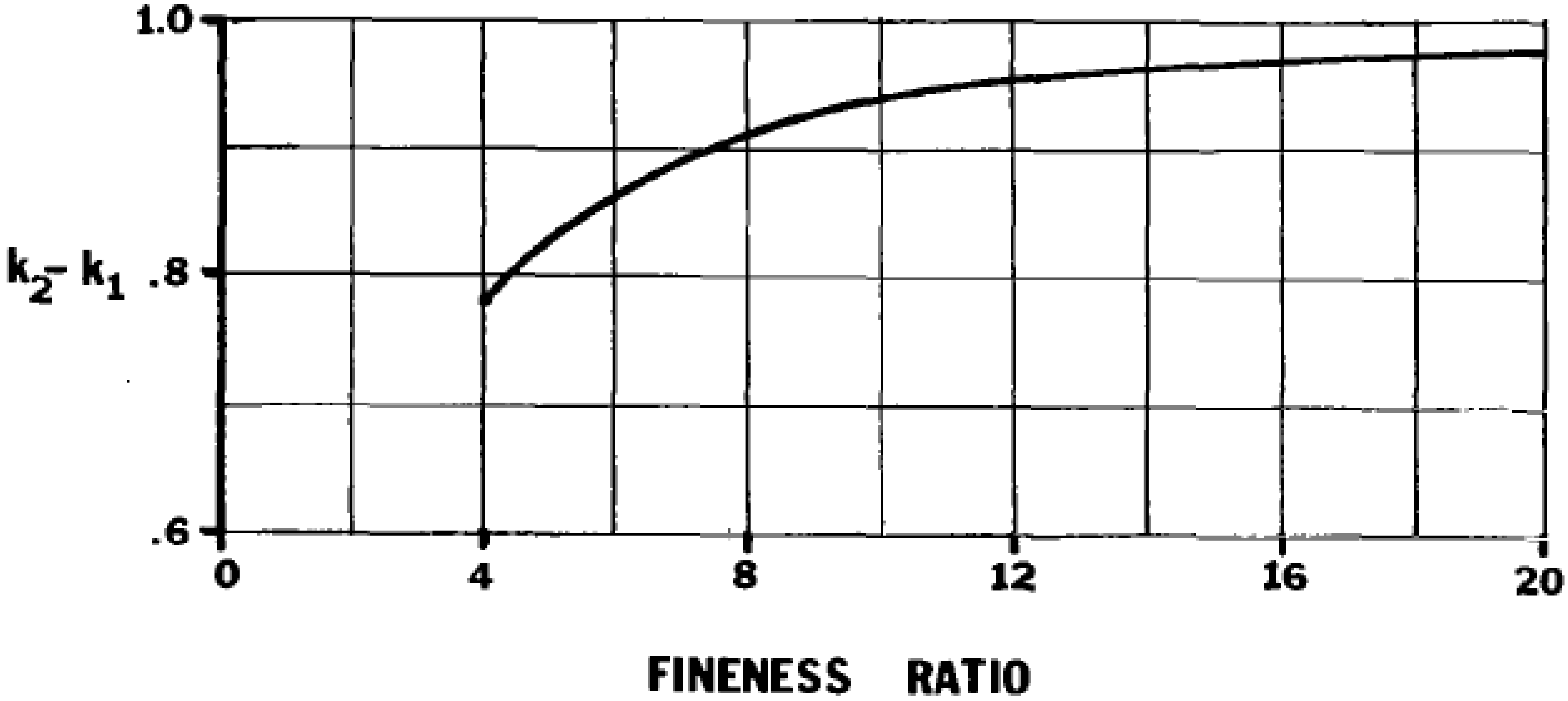


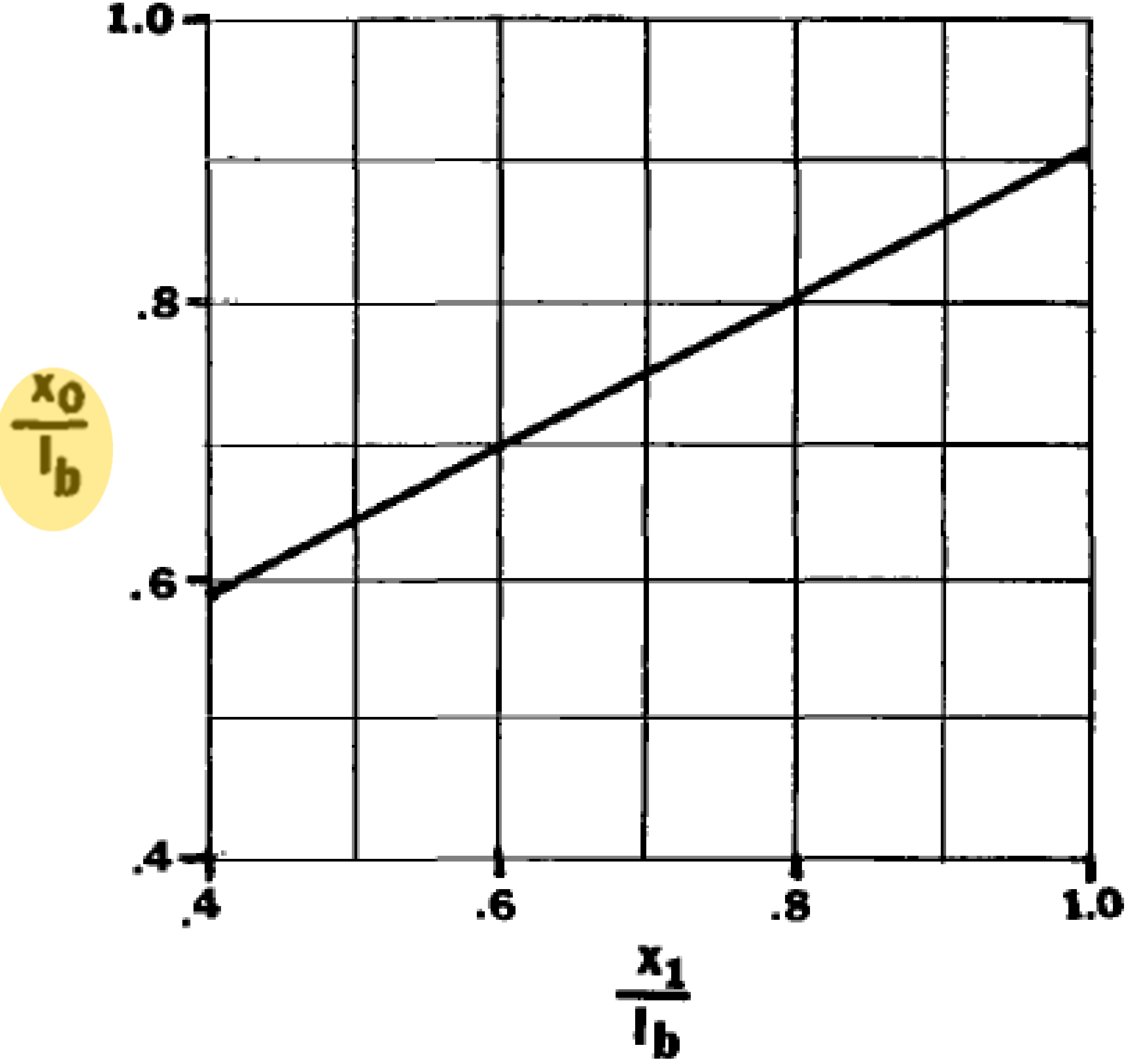
Figure 7. Body station where flow becomes viscous.

Fig A5



Ref: Smetana (Fig 6)

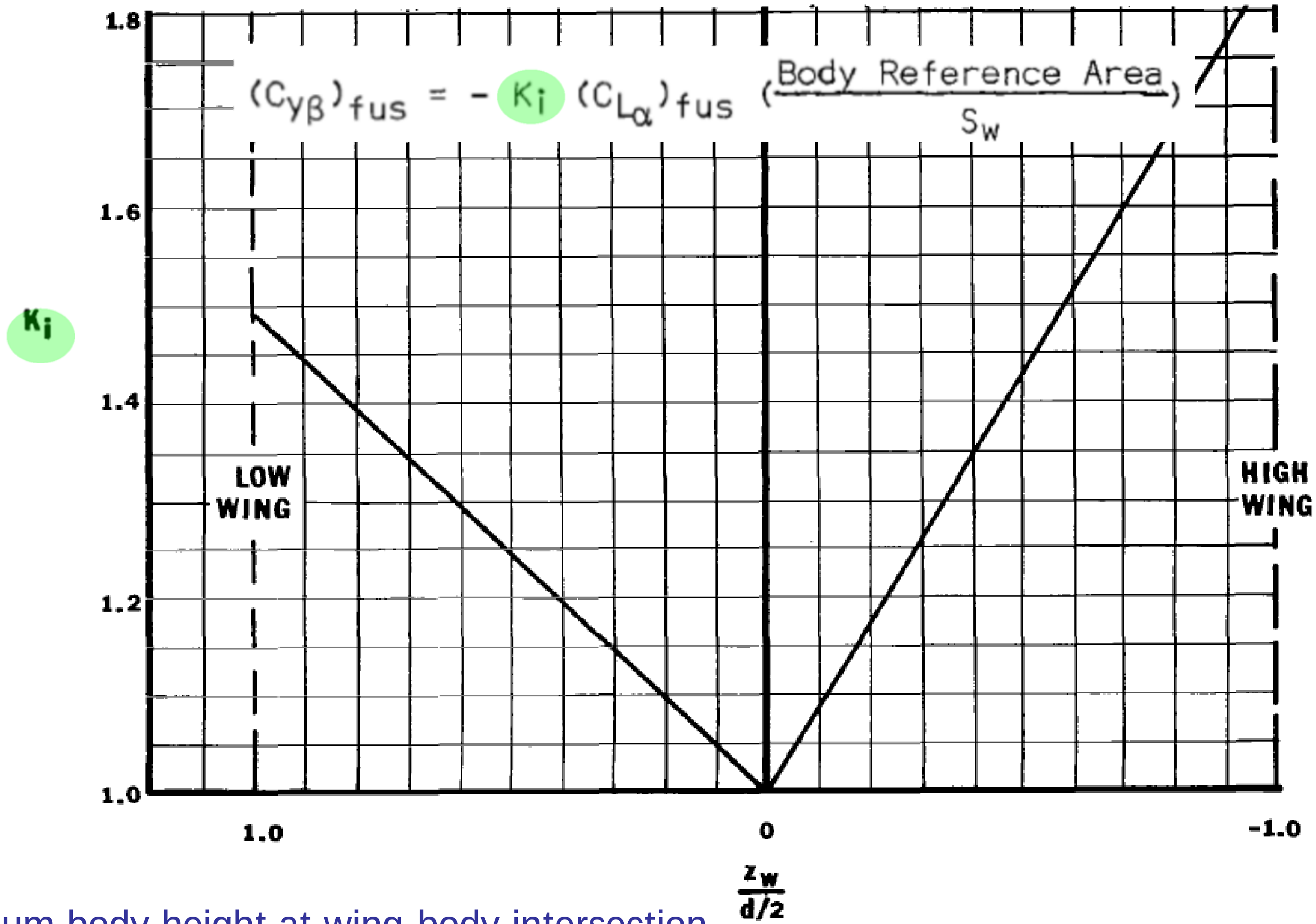
Fig A6



Ref: Smetana (Fig 7)

Fig B6

Ref: Smetana (Fig 23)



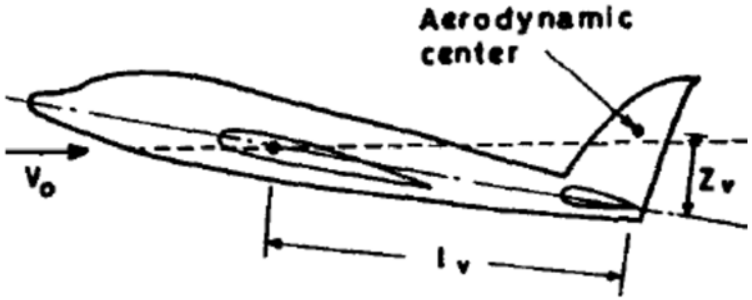
d = maximum body height at wing-body intersection

z_w = distance from body centerline to quarter-chord point of exposed wing root chord (positive for the quarter-chord point below the body centerline) ,

Lateral-Directional C_{Y_β}

Método I

Tail contribution.



$$C_{y\beta,V} = -k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v \left(\frac{S_v}{S}\right)$$

$$\left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v = 0.724 + \frac{3.06 S_v/S}{1+\cos\Lambda_{c/4}} + \frac{0.4 z_w}{d_{f,\max}} + 0.009A$$

$$a_v = a_w = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{k^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{\beta^2}\right) + 4}}$$

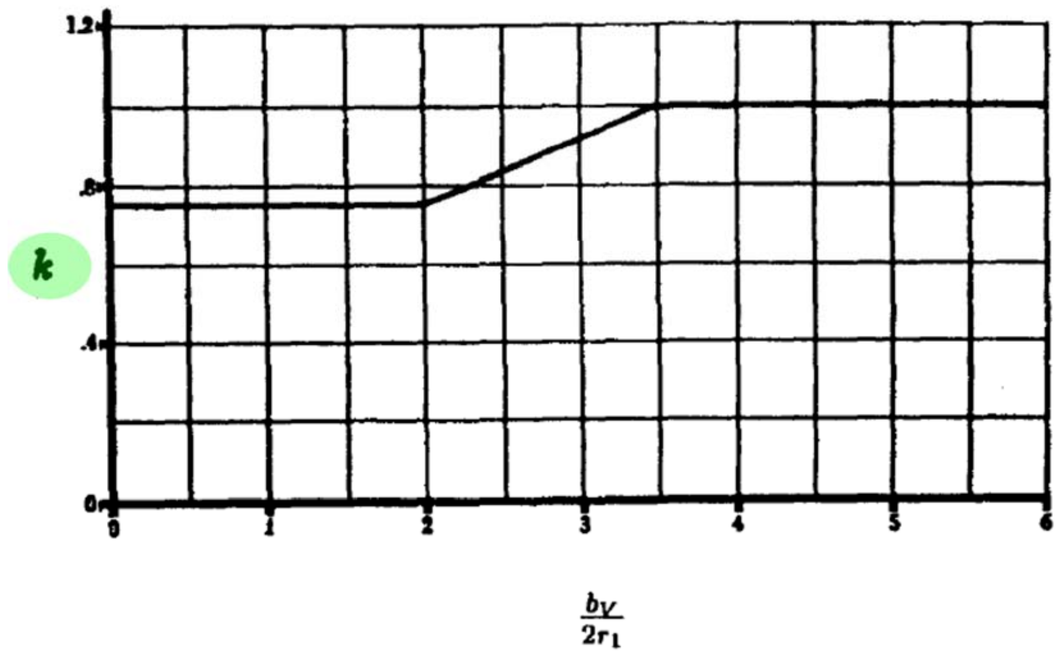


Fig B5

Ref: Pamadi

Lateral-Directional C_{L_β}

Método I

Método Alternativo Cálculo completo C_{L_β} (Smetana)

Ref: Smetana

ala-fuselaje

No diedro

$$(C_{\ell_\beta})_{total} = (C_{\ell_\beta})_W + (C_{\ell_\beta})_V + (\Delta C_{\ell_\beta})_1 + (\Delta C_{\ell_\beta})_2 + (C_{\ell_\beta})_{W,\Gamma=0}$$

ala vertical ala-vertical

Contribución ala

$$(C_{\ell_\beta})_W = \left(\frac{C_{\ell_\beta}}{\Gamma}\right)\Gamma + (\Delta C_{\ell_\beta})_{tip\ shape}$$

Fig B15

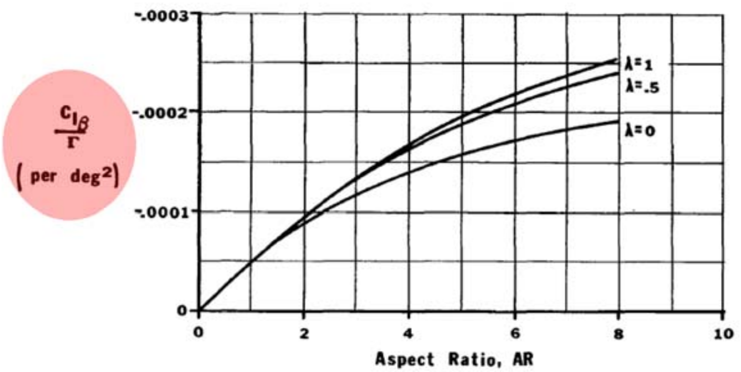


Fig B14

Figure 26. Values for (C_{ℓ_β}/Γ) for various aspect and taper ratios.

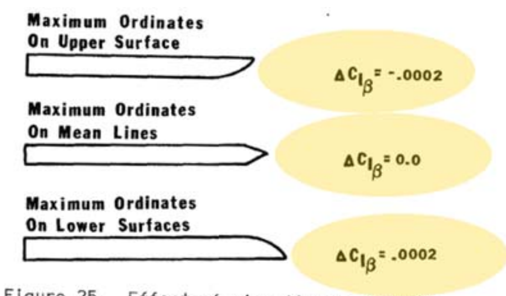
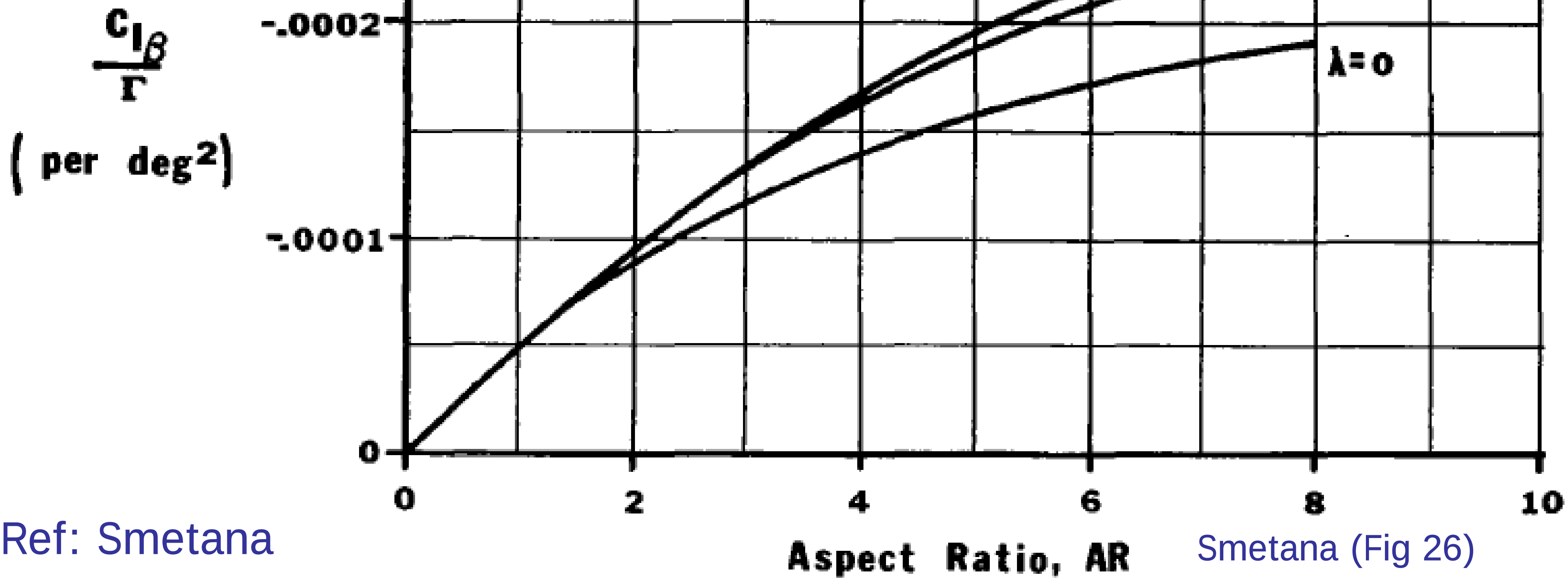


Figure 25. Effect of wing tip shape on C_{ℓ_β} per radian.

Fig B14

Hay que convertir a $\frac{1}{rad}$. Como a $\left(\frac{C_{l_\beta}}{\Gamma}\right)$ tiene unidades $\frac{1}{\deg^2}$, una vez que se multiplica por el diedro en grados: $\left(\frac{C_{l_\beta}}{\Gamma}\right)\Gamma$ y se corrige a radianes

$$\left(\frac{C_{l_\beta}}{\Gamma}\right)\Gamma \times \frac{180}{\pi}$$

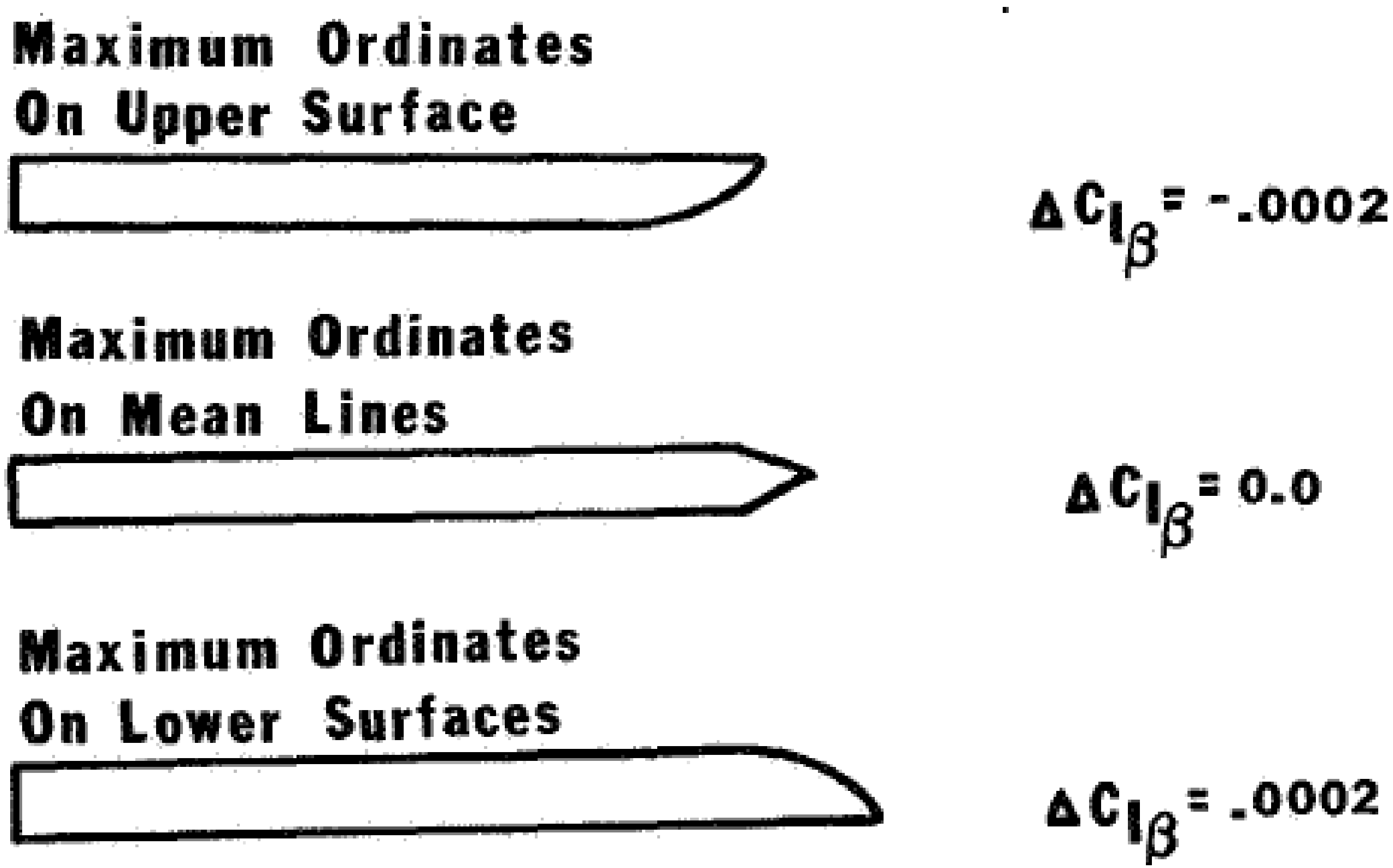


Smetana (Fig 26)

Ref: Smetana

Fig B15

Corrección C_{l_β} en función de la terminación de la punta del ala



Ref: Smetana

Smetana (Fig 25)

Lateral-Directional C_{L_β}

Método I

Alternative method

$$(C_{\ell_\beta})_{total} = (C_{\ell_\beta})_W + (C_{\ell_\beta})_V + (\Delta C_{\ell_\beta})_1 + (\Delta C_{\ell_\beta})_2 + (C_{\ell_\beta})_{W,\Gamma=0}$$

A theoretical study for unswept ,elliptical wings with zero dihedral (NASA TR-1269)

$$(C_{\ell_\beta})_{W,\Gamma=0} = C_L\left[-\frac{16}{3\pi^2 AR} + .05\right] \text{ per radian}$$

O

$$(C_{\ell_\beta})_{W,\Gamma=0} = C_L\left[-\frac{k(.71\lambda + .29)}{AR\,\lambda} + .05\right] \text{ per radian}$$

k = 1.0 for straight wing tips
k = 1.5 for round wing tips

λ – estrechamiento y AR – Alargamiento ala

Contribución vertical

$$(C_{\ell_\beta})_V = -a_V \frac{S_V}{S_W}\frac{z_V}{b_W}\eta_V = -(C_{n_\beta})_V \frac{z_V}{\ell_V}$$

z_V = distance from the center of pressure of the vertical tail to the airplane's x-axis (positive for vertical tail above the x-axis).

Contribución ubicación ala

	Wing-fuselage $(\Delta C_{\ell_\beta})_1$	Wing-vertical Tail $(\Delta C_{\ell_\beta})_2$
High Wing	-.0006	.00016
Mid Wing	0	0
Low Wing	.0008	-.00016

Ref: Smetana

Contribución Ala - I

C_{L_β}

Método II

ala, horizontal, V-tail

$$C_{l\beta} = (C_{l\beta})_{W(B)} + (C_{l\beta})_V + \left(C_{l_\beta}\right)_h + \left(C_{l_\beta}\right)_C + \left(C_{l_\beta}\right)_{vee}$$

vertical, canard

Contribución canard,horizontal y V-tail muy pequeñas → despreciable en 1ª aproximación

Estimation of combined wing contribution. Aproximación subsonic speeds

$$(C_{l\beta})_{W(B)} = C_L\left[\left(\frac{C_{l\beta}}{C_L}\right)_{\Lambda_{c/2}} K_{M\Lambda}K_f + \left(\frac{C_{l\beta}}{C_L}\right)_A\right] + \Gamma\left[\frac{C_{l\beta}}{\Gamma}K_{M\Gamma} + \frac{\Delta C_{l\beta}}{\Gamma}\right] + (\Delta C_{l\beta})_{z_w}$$

the dihedral angle Γ is in deg. Asegurarse que las unidades finales son 1/rad

A is the theoretical wing aspect ratio,
d is the average fuselage diameter at the wing root,
b is the wing span,
z_W is vertical distance between the fuselage centerline and wing root quarter chord point, positive if the wing is below the fuselage centerline.

Ref: Pamadi

$$\frac{\Delta C_{l\beta}}{\Gamma} = -0.0005\sqrt{A}\left(\frac{d}{b}\right)^2$$ $1/deg$

$$(\Delta C_{l\beta})_{z_w} = \frac{1.2\sqrt{A}}{57.3}\left(\frac{z_w}{b}\right)\left(\frac{2d}{b}\right)$$ $1/deg$

Contribución Ala - II

C_{L_β}

Ref: Pamadi

$$(C_{l\beta})_{W(B)} = C_L\left[\left(\frac{C_{l\beta}}{C_L}\right)_{\Lambda_{c/2}} K_{M\Lambda}K_f + \left(\frac{C_{l\beta}}{C_L}\right)_A\right] + \Gamma\left[\frac{C_{l\beta}}{\Gamma}K_{M\Gamma} + \frac{\Delta C_{l\beta}}{\Gamma}\right] + (\Delta C_{l\beta})_{z_w}$$

Fig B13

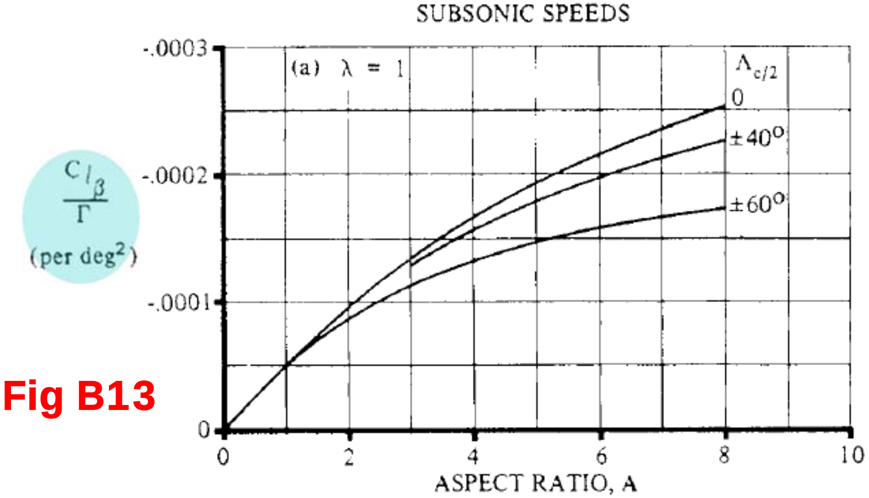
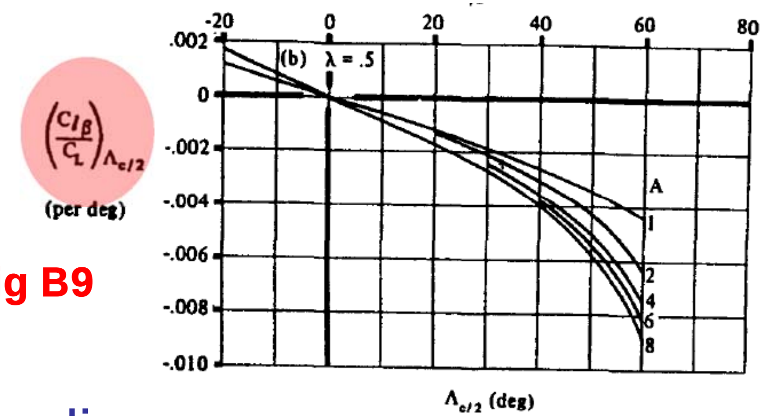


Fig B9



Ref: Pamadi

Fig B10

Fig B11

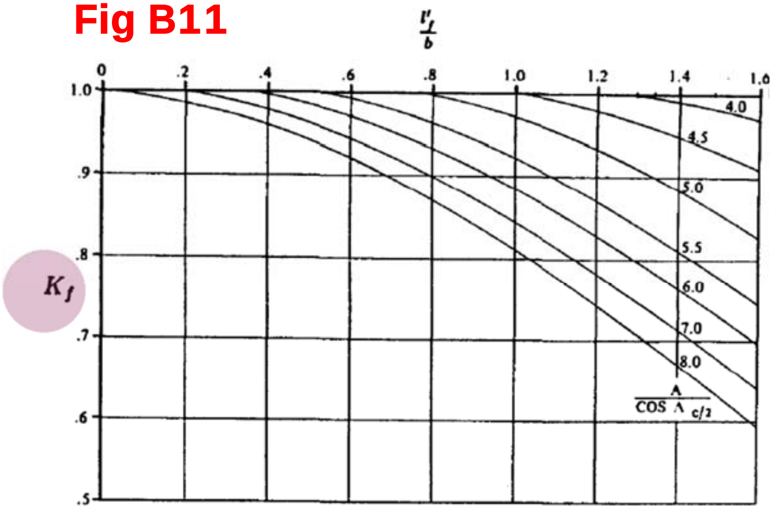


Fig. 3.98 Fuselage correction factor K_f (Ref. 1).

Fig B12

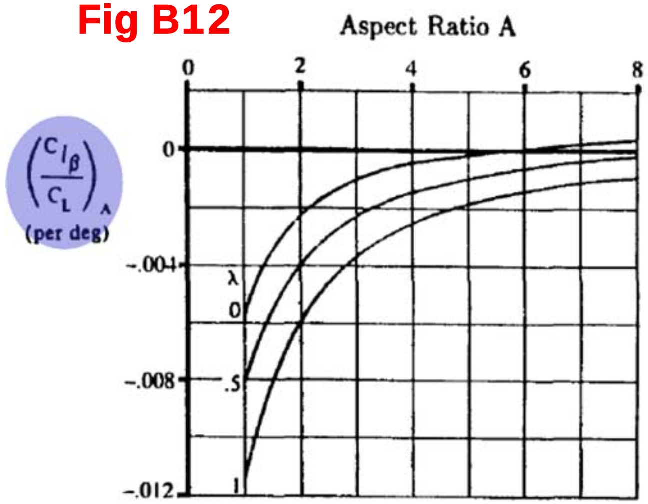


Fig. 3.99 Aspect ratio contribution to $C_{l\beta}$ (Ref. 1).

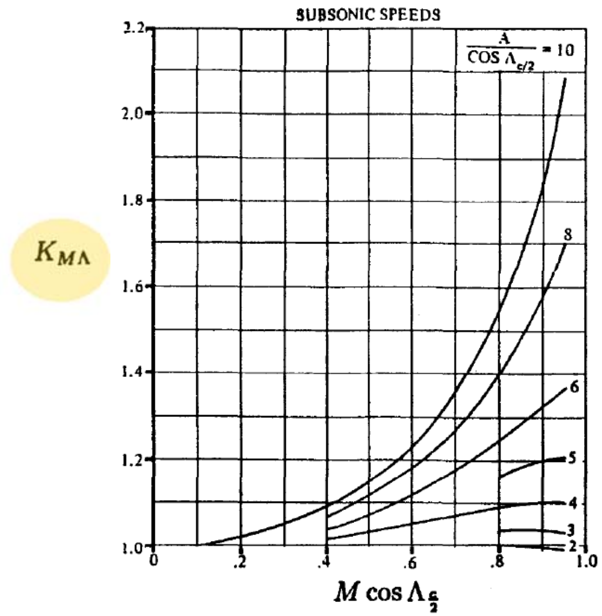


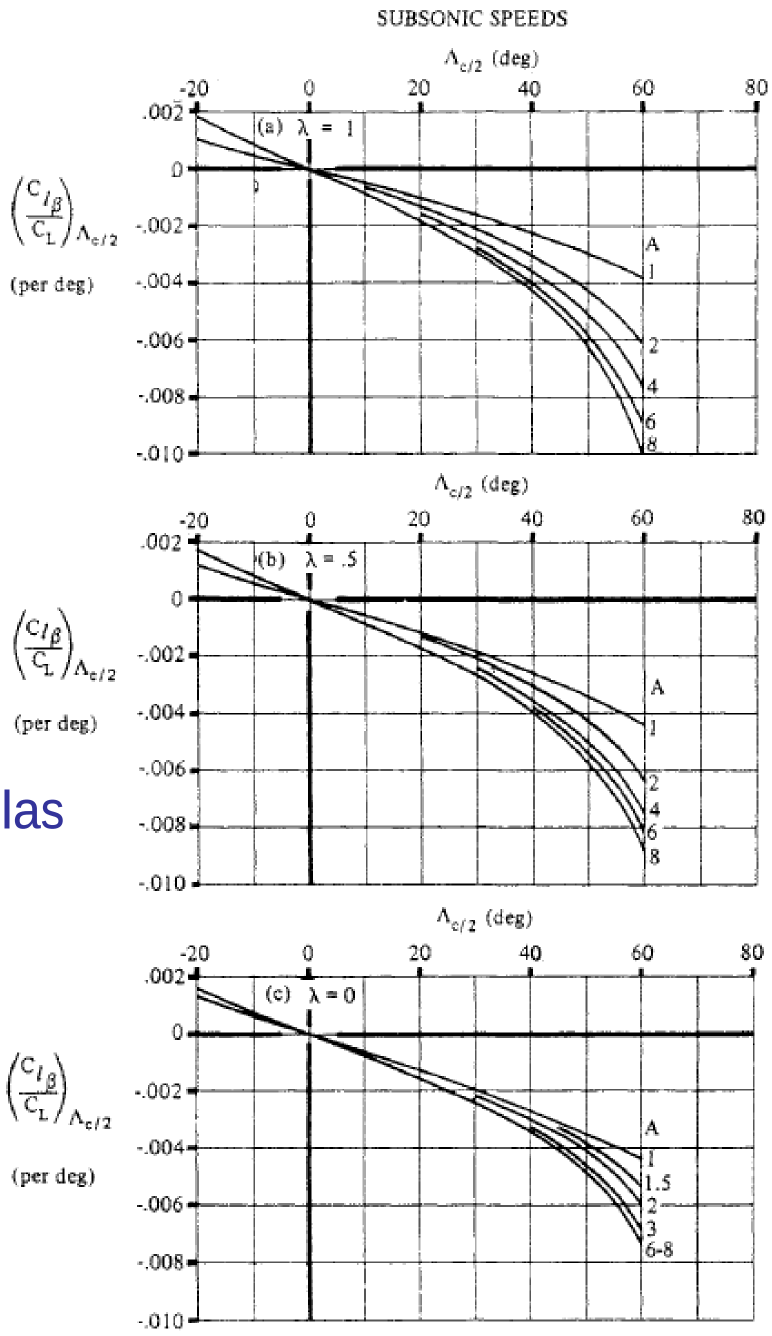
Fig. 3.97 Compressibility correction factor $K_{M\Lambda}$ (Ref. 1).

Fig B9

Seleccionar curvas que más se acerquen
En función de estrechamiento (λ) y
de la flecha $\Lambda_{c/2}$

Derivadas en 1/deg, por lo que hay que convertirlas

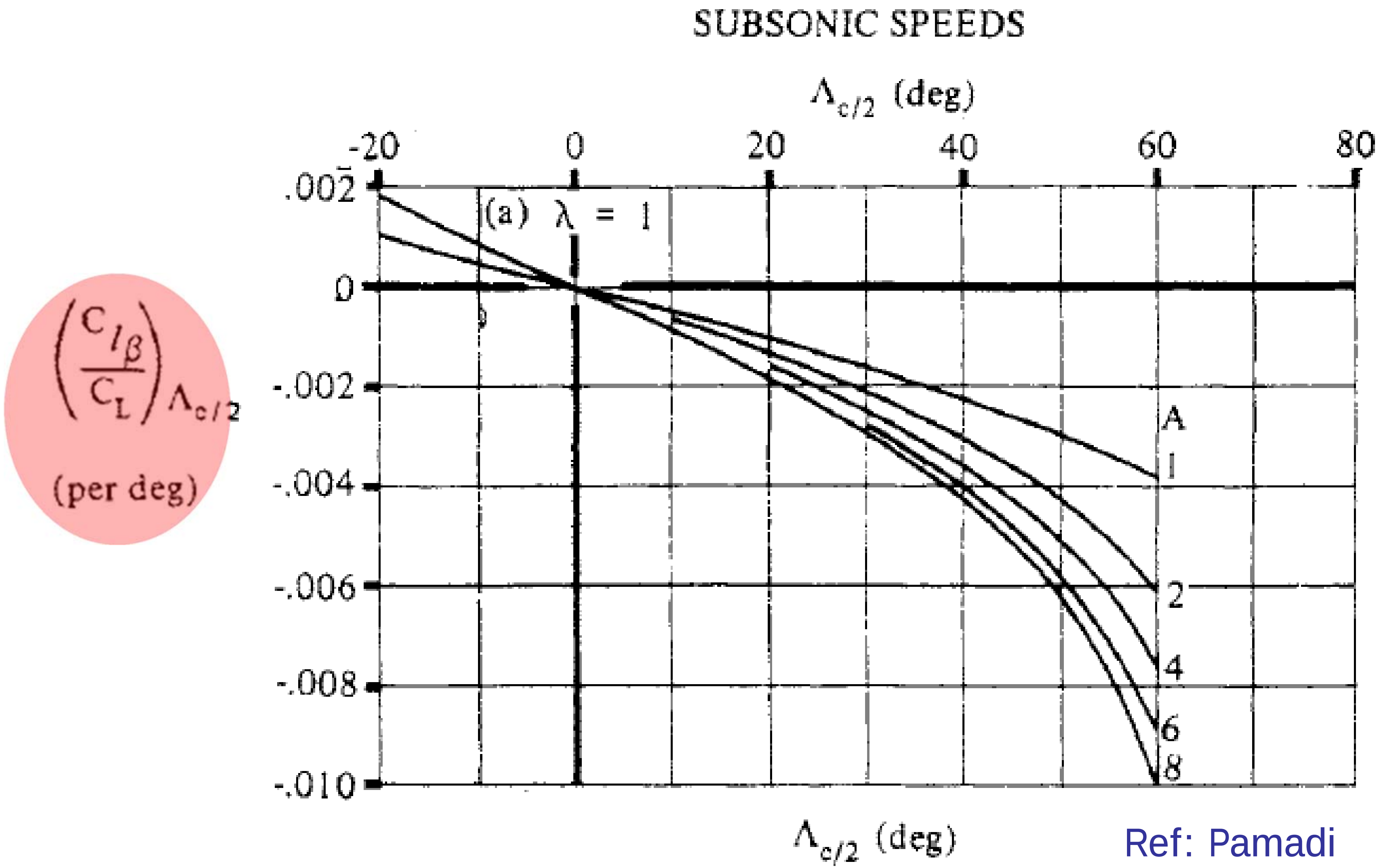
Fig. 3.96 Wing-sweep contribution to $C_{l\beta}$ (Ref. 1).



Ref: Pamadi

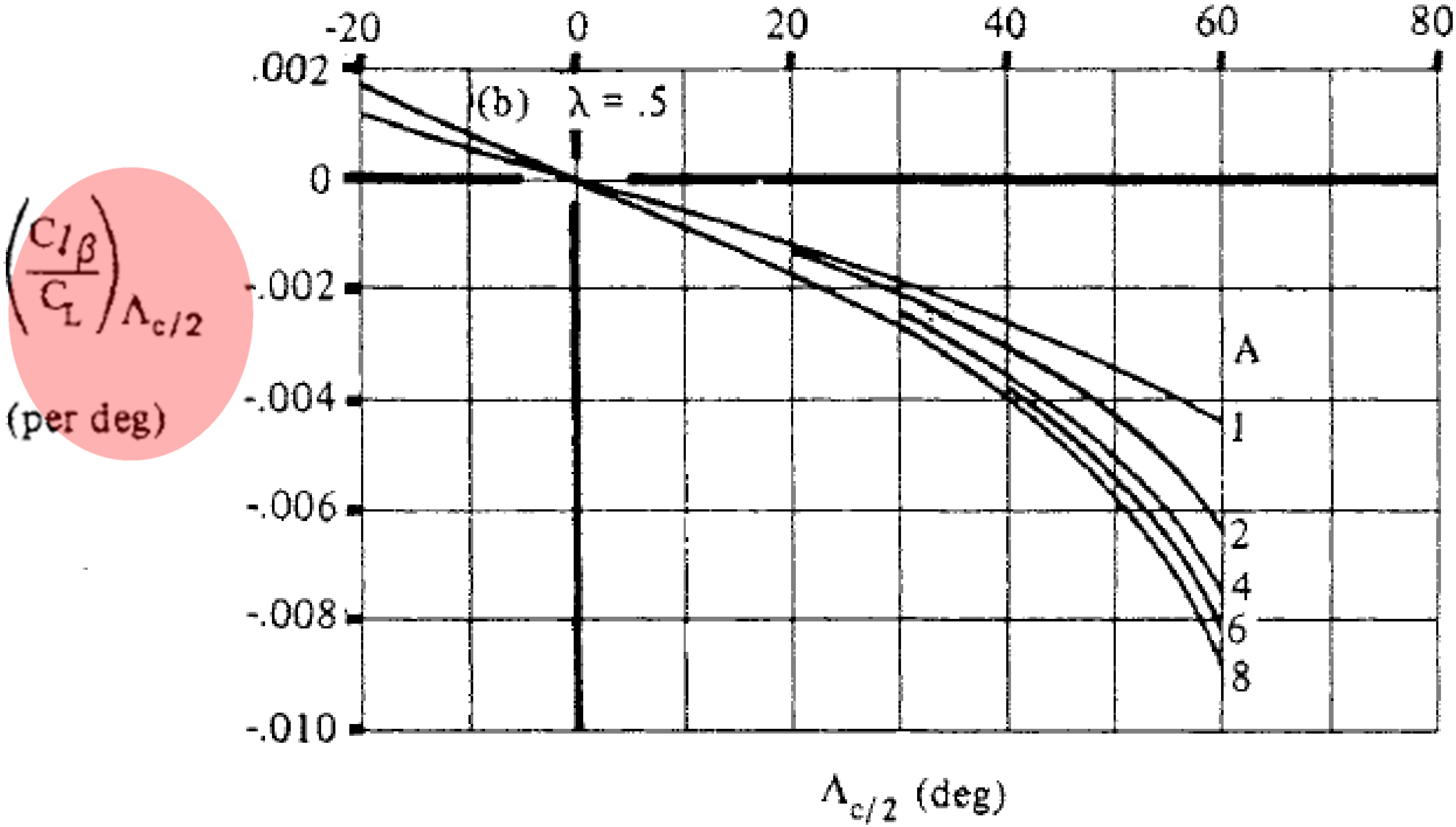
Fig. 3.96 Wing-sweep contribution to $C_{l\beta}$ (Ref. 1).

Fig B9 – Cont. I



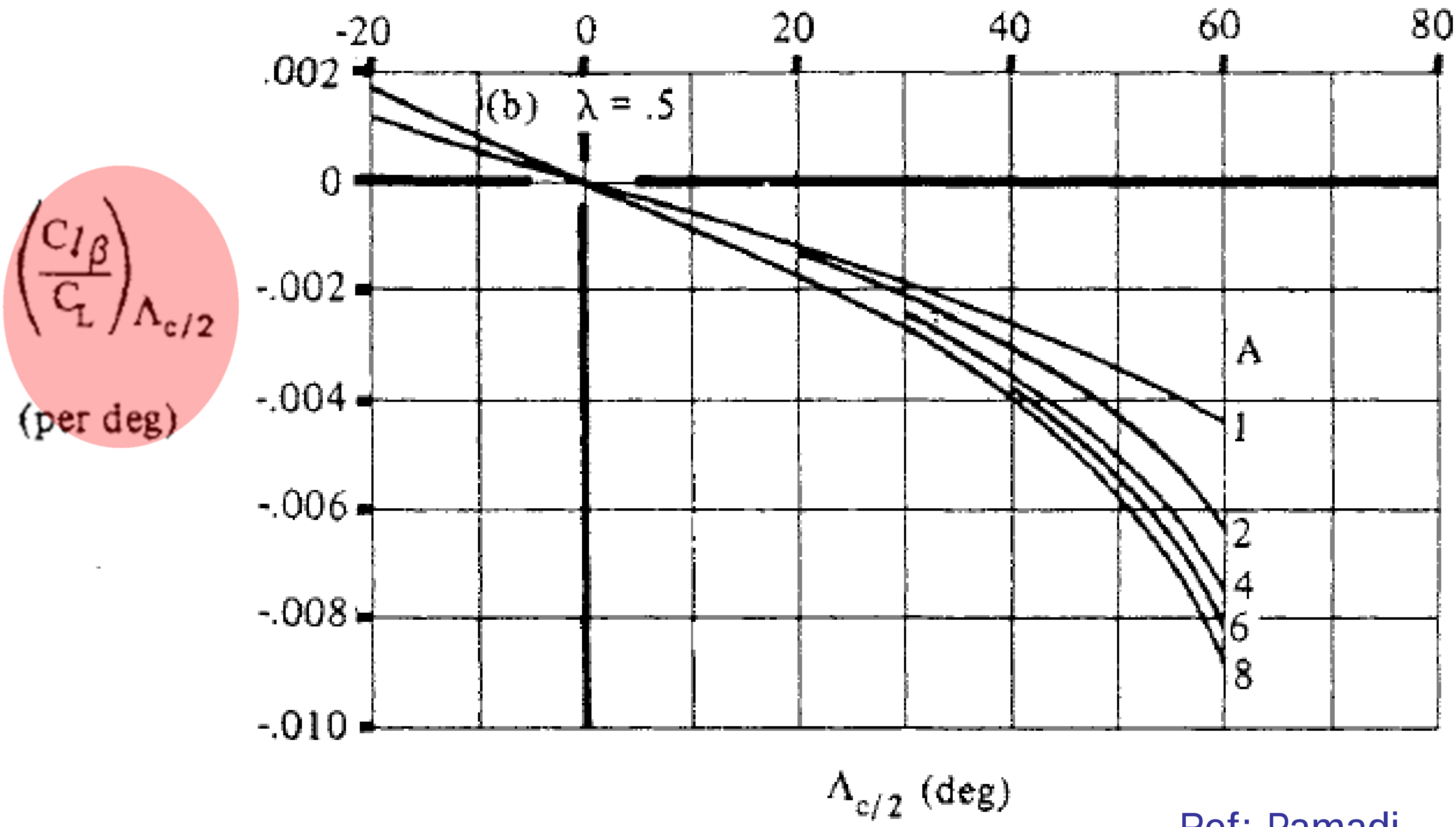
Ref: Pamadi

Fig B9 – Cont. II



Ref: Pamadi

Fig B9 – Cont. III



Ref: Pamadi

Fig B10

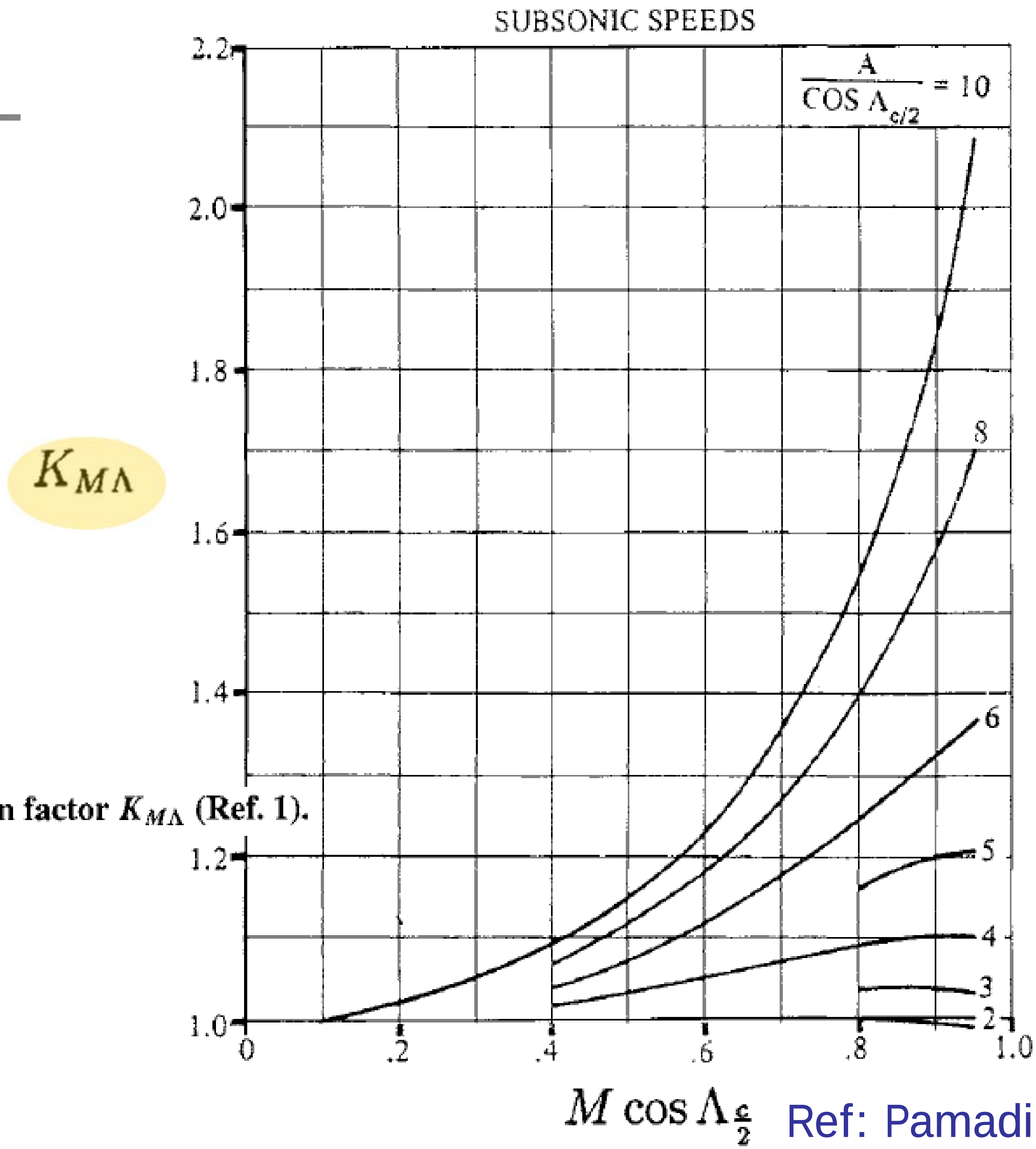
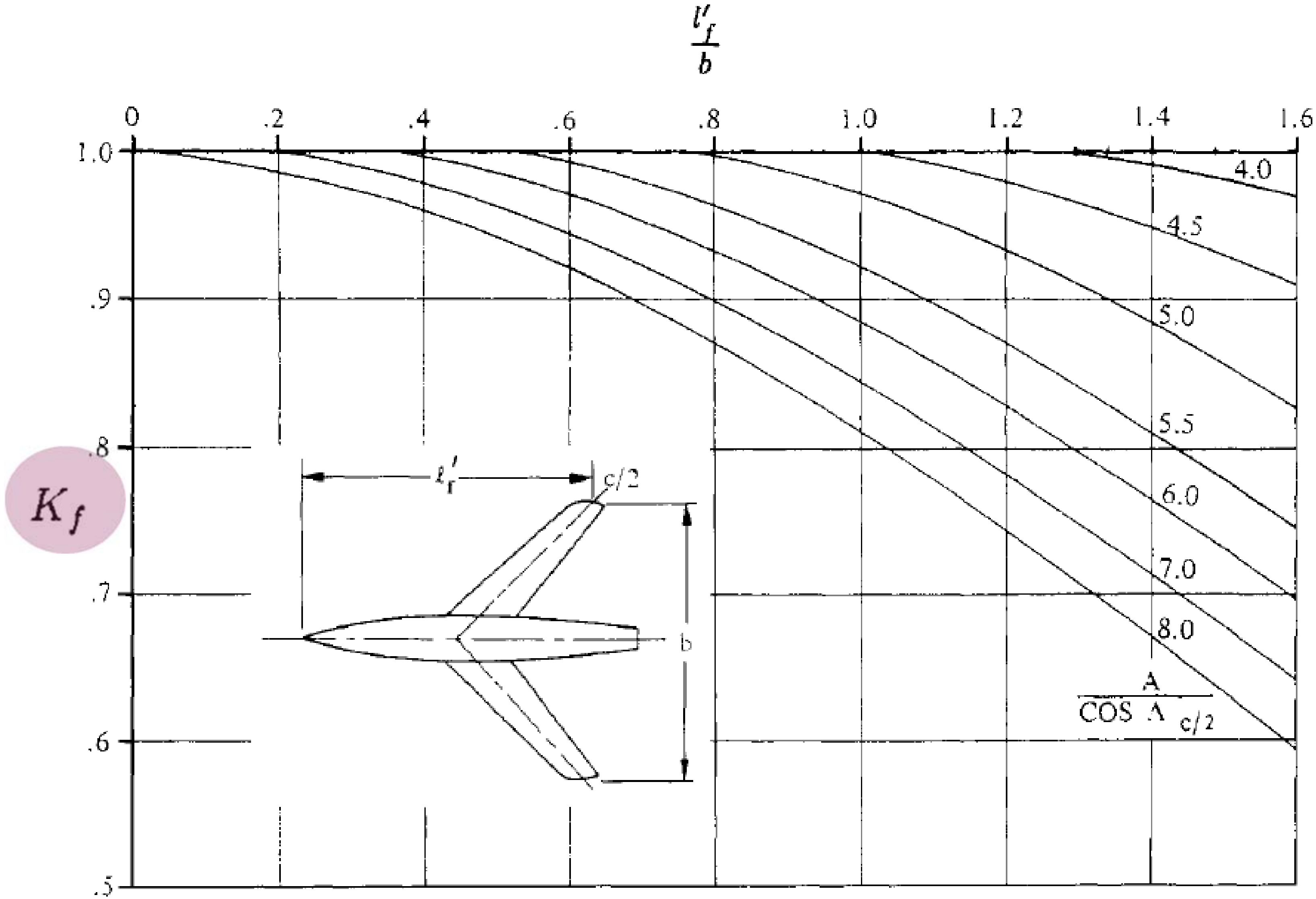


Fig. 3.97 **Compressibility correction factor $K_{M\Lambda}$ (Ref. 1).**

Ref: Pamadi

Fig B11

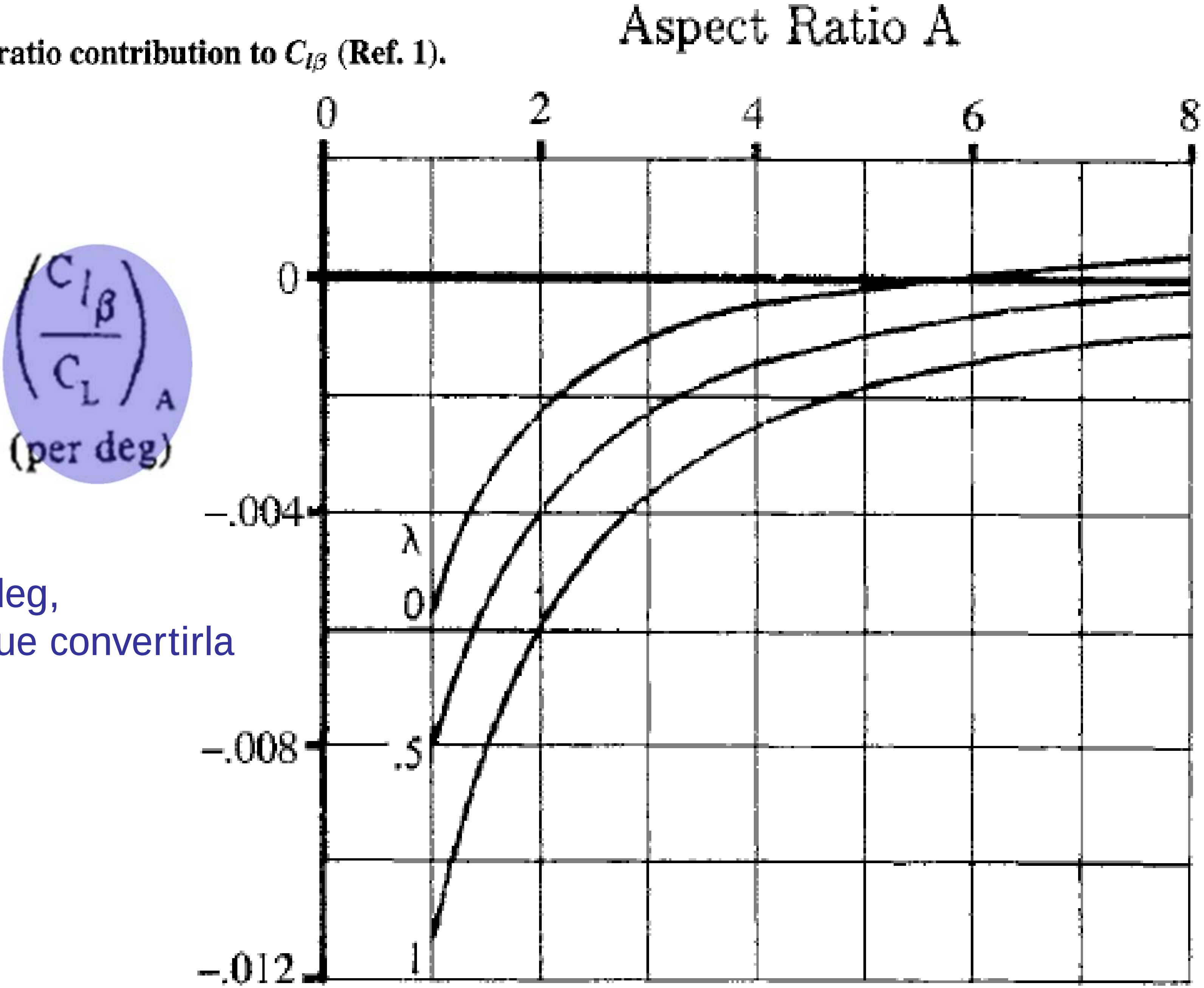


Ref: Pamadi

Fig. 3.98 Fuselage correction factor K_f (Ref. 1).

Fig B12

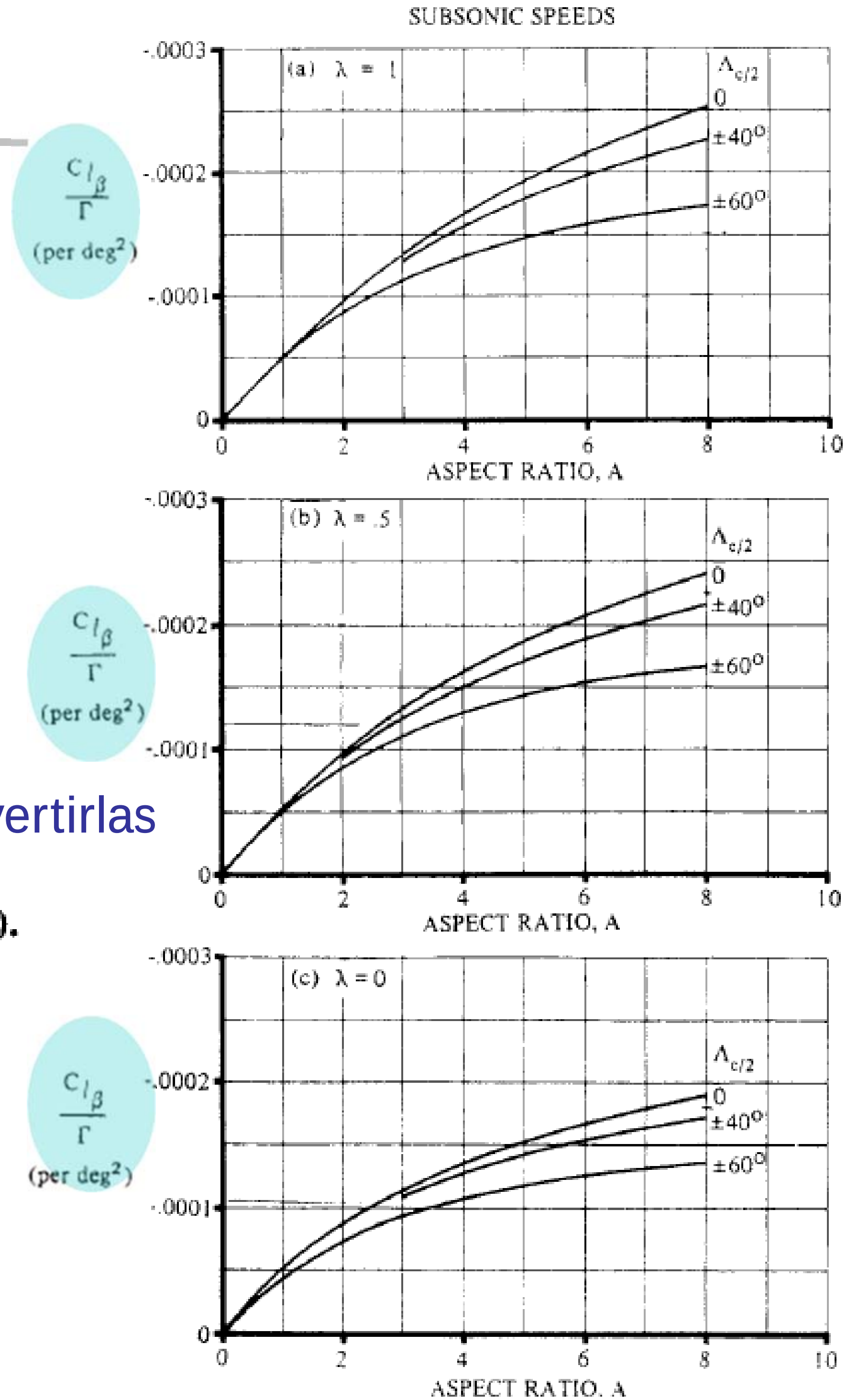
Fig. 3.99 Aspect ratio contribution to $C_{l\beta}$ (Ref. 1).



Derivadas en 1/deg,
por lo que hay que convertirla

Ref: Pamadi

Fig B13



Seleccionar curvas que más se acerquen
En función de estrechamiento (λ) y
de la flecha $\Lambda_{c/2}$

Derivadas en 1/deg, por lo que hay que convertirlas

Fig. 3.100 Contribution of wing dihedral to $C_{l\beta}$ (Ref. 1).

Ref: Pamadi

Fig B13 – Cont. I

Ref: Pamadi

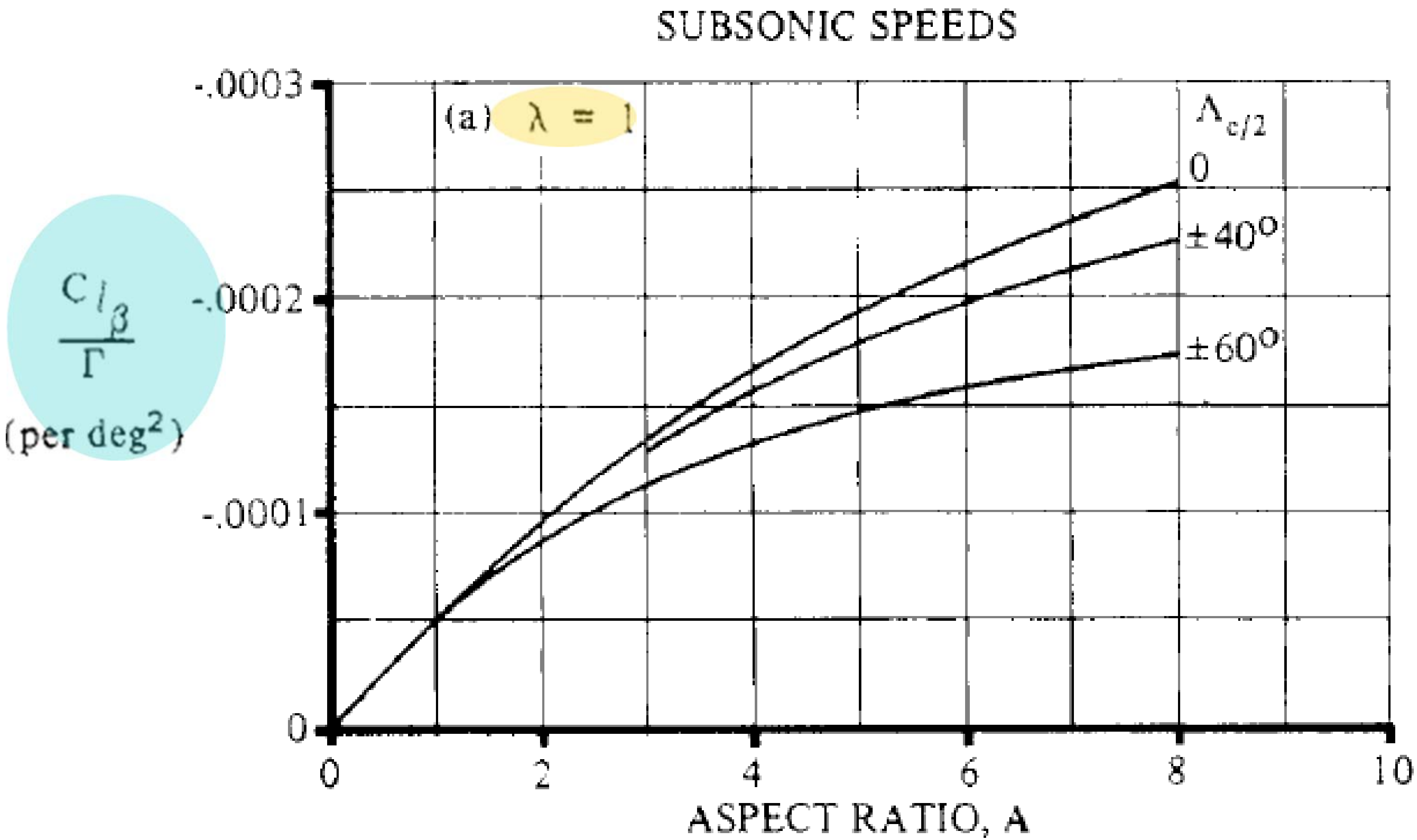


Fig B13 – Cont. II

Ref: Pamadi

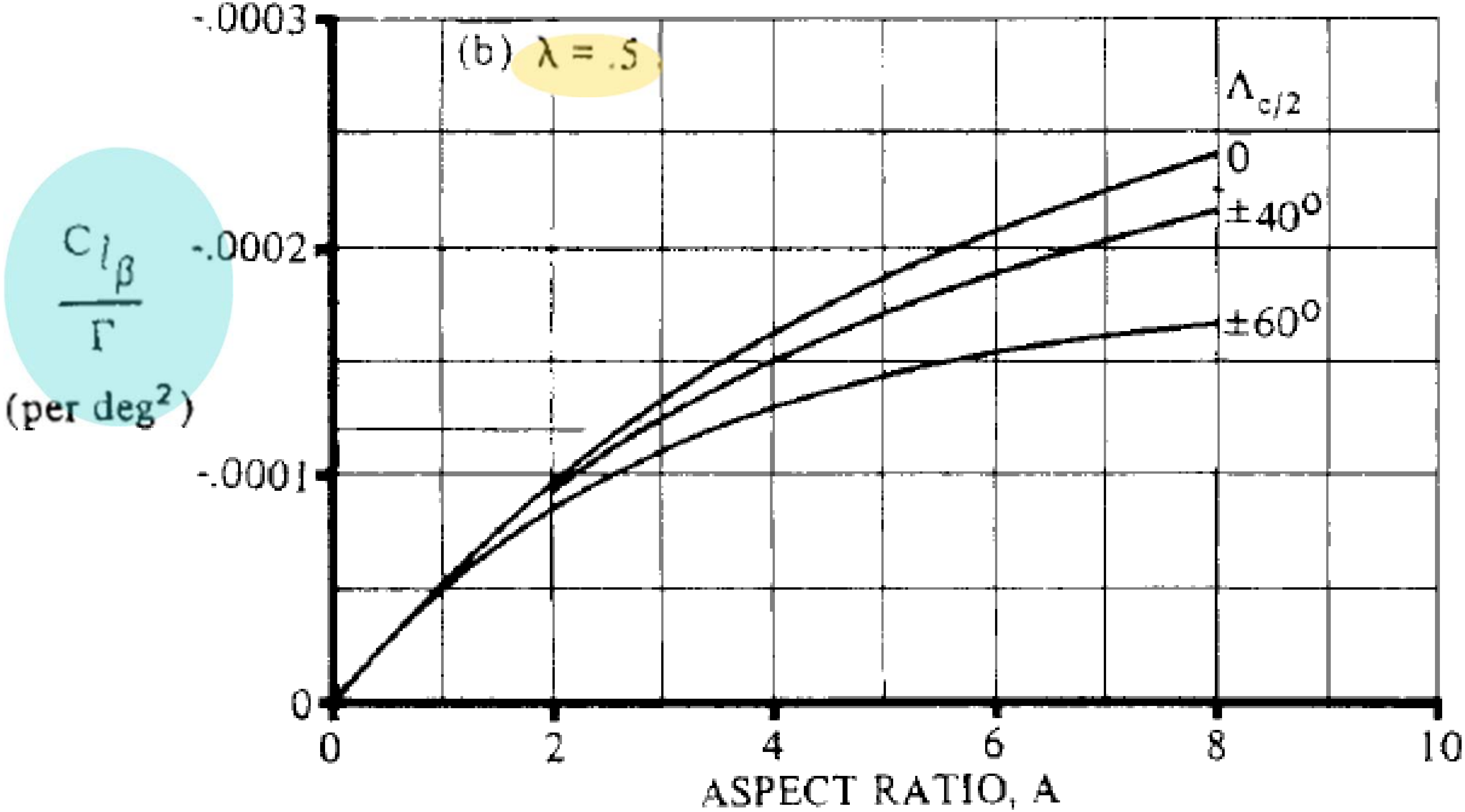


Fig B13 – Cont. III

Ref: Pamadi

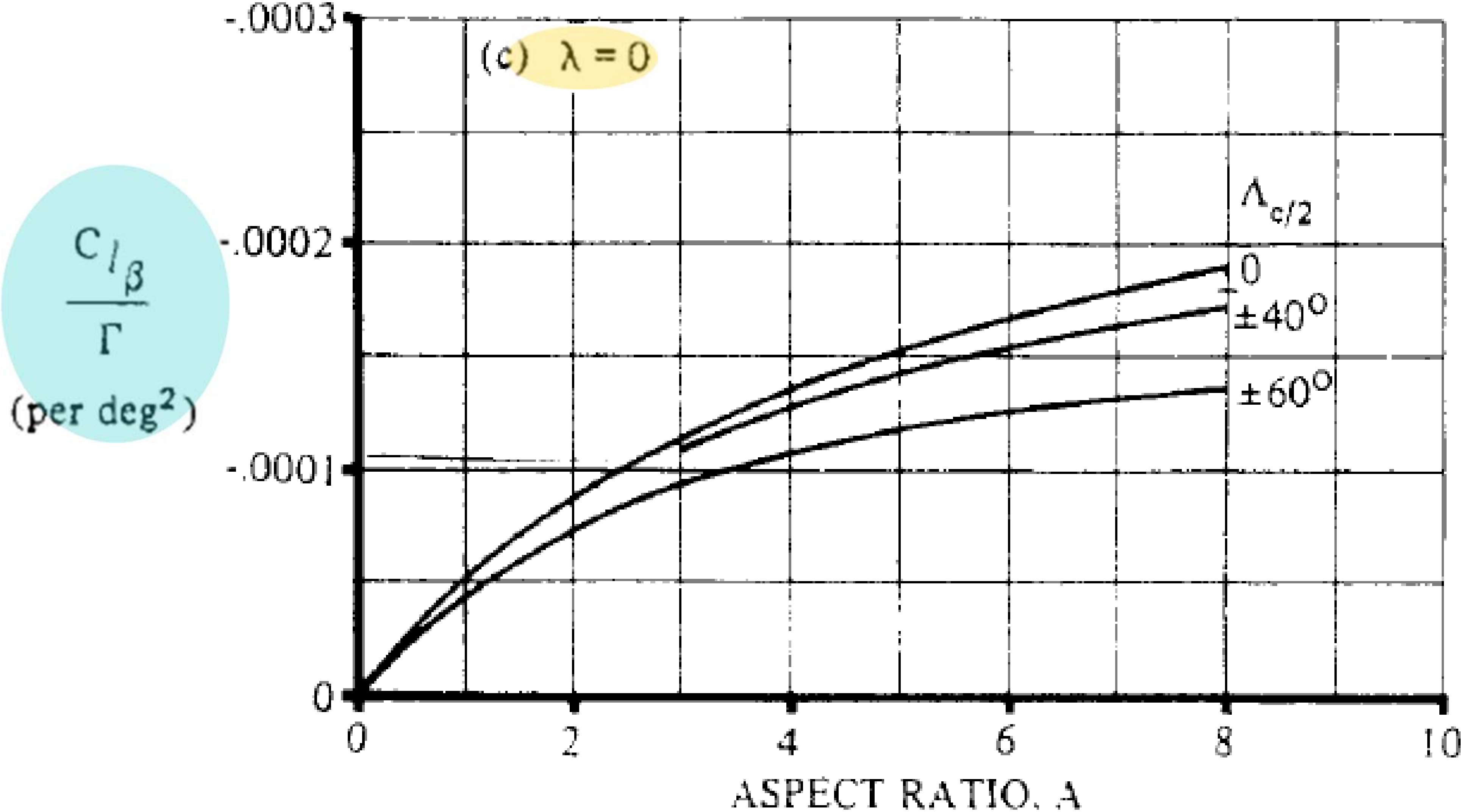


Fig B14

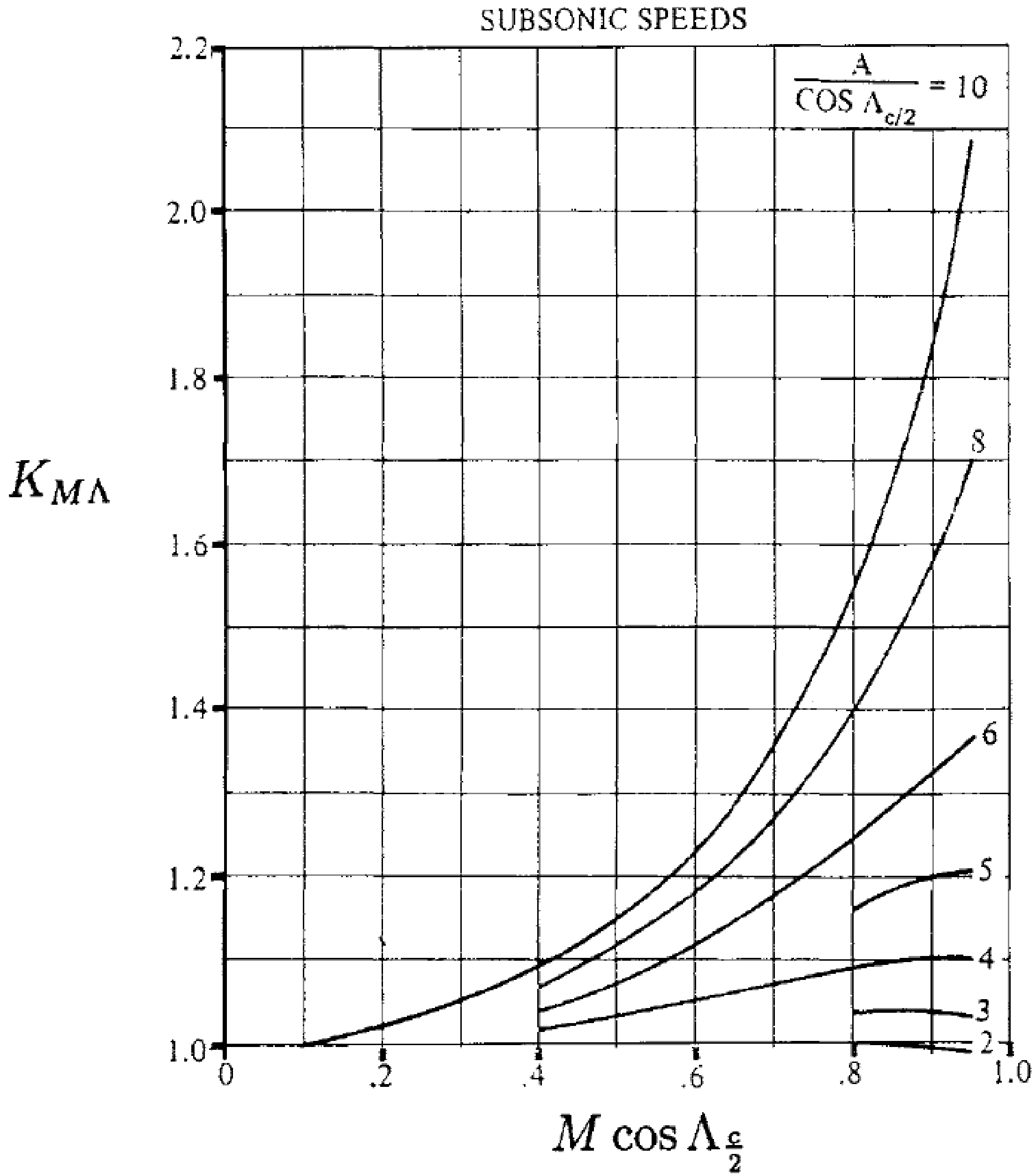


Fig. 3.97 **Compressibility correction factor $K_{M\Lambda}$ (Ref. 1).**

Fig B15

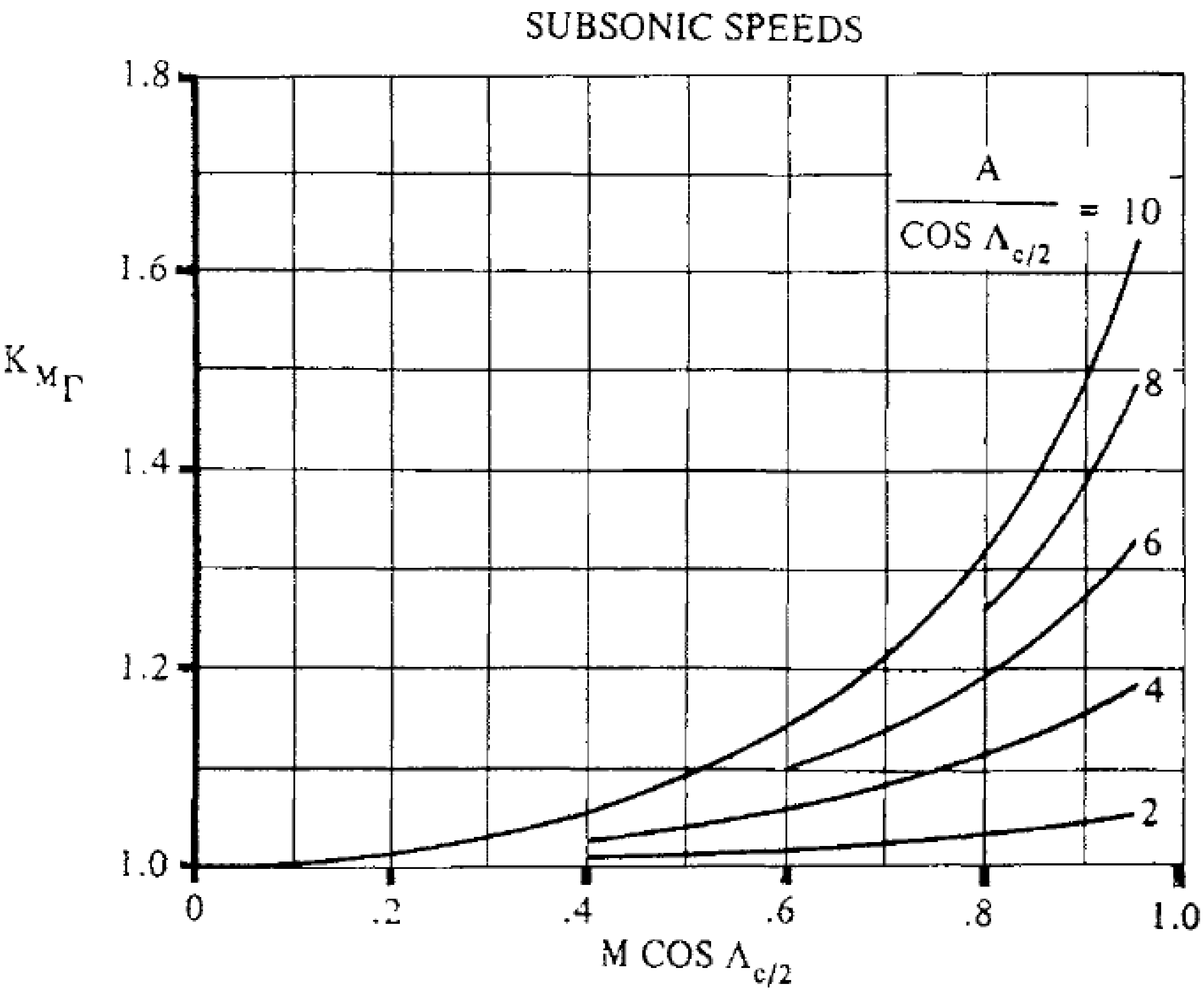
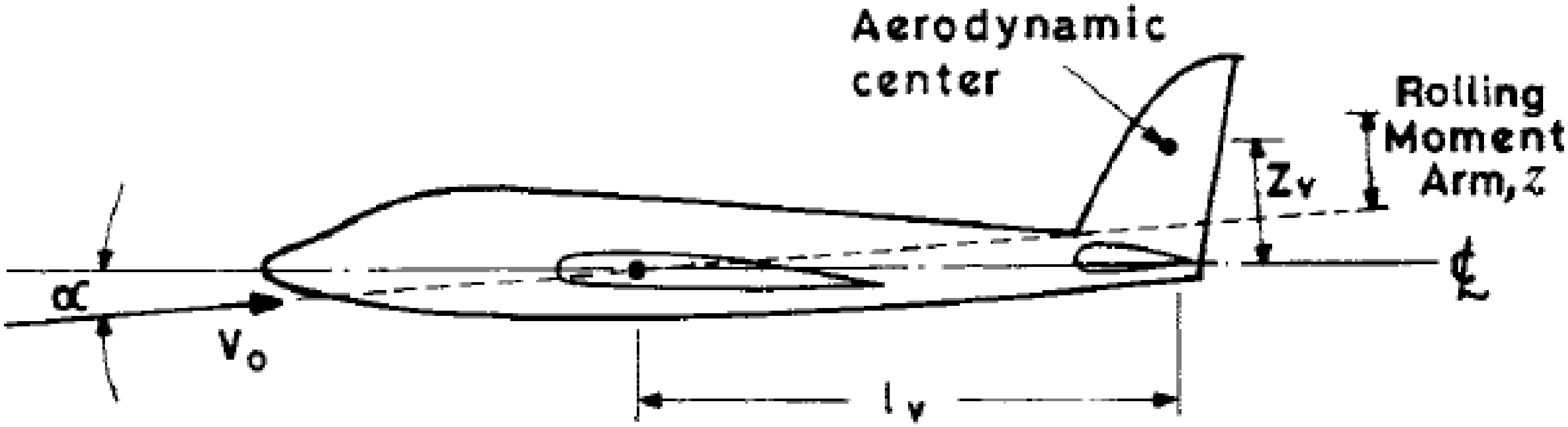
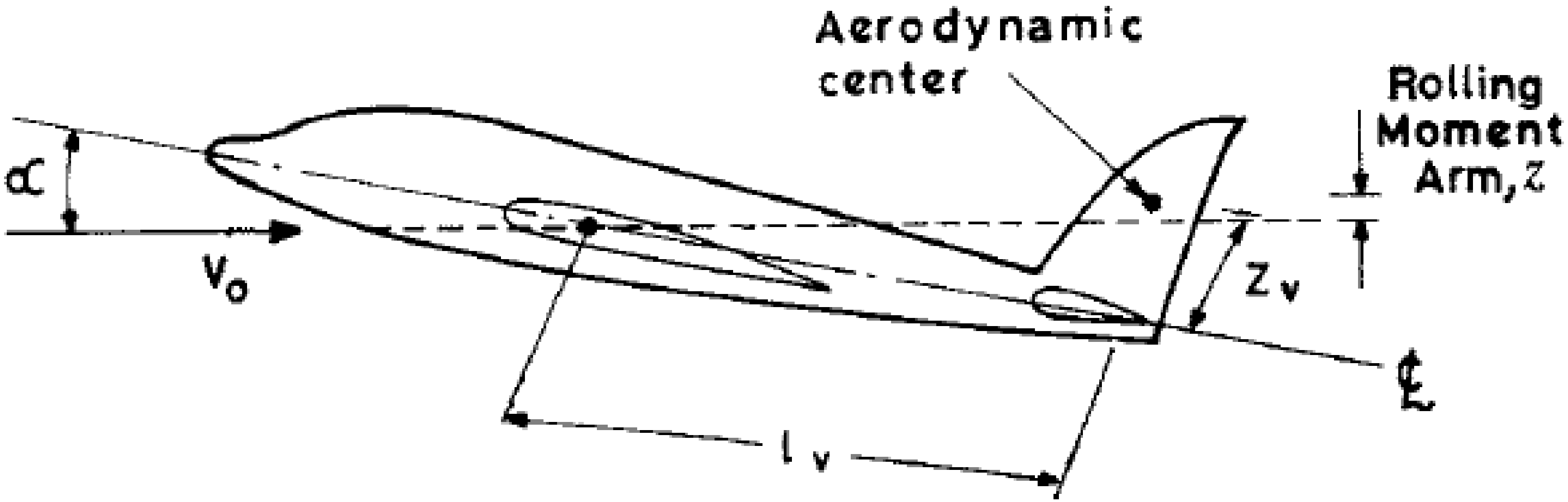


Fig. 3.101 Compressibility correction to wing dihedral effect.[1]



a) Low angle of attack

Ref: Pamadi



b) High angle of attack

Fig. 3.102 Effect of angle of attack on vertical tail rolling moment arm.

Contribución Vertical C_{L_β}

The vertical tail contribution

Método I

$$(C_{l\beta})_V = C_{y\beta,V}\left(\frac{z}{b}\right)$$

$$z = z_v \cos\alpha - l_v \sin\alpha$$

$$C_{y\beta,V} = -ka_v\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v\left(\frac{S_v}{S}\right)$$

$$(C_{l\beta})_V = -ka_v\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v\left(\frac{S_v}{S}\right)\left(\frac{z_v\cos\alpha - l_v\sin\alpha}{b}\right)$$

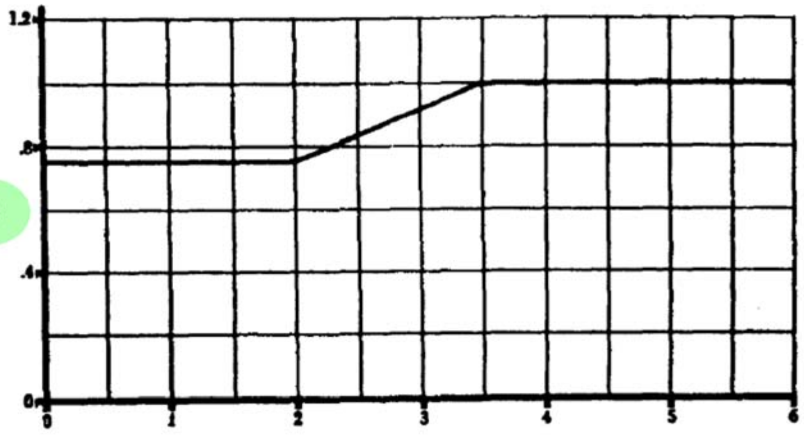
Ref: Pamadi

$$\left(1+\frac{\partial\sigma}{\partial\beta}\right)\eta_v = 0.724 + \frac{3.06 S_v/S}{1+\cos\Lambda_{c/4}} + \frac{0.4 z_w}{d_{f,\max}} + 0.009A$$

$$a_v = a_w = \frac{2\pi A}{2+\sqrt{\frac{A^2\beta^2}{k^2}\left(1+\frac{\tan^2\Lambda_{c/2}}{\beta^2}\right)+4}}$$

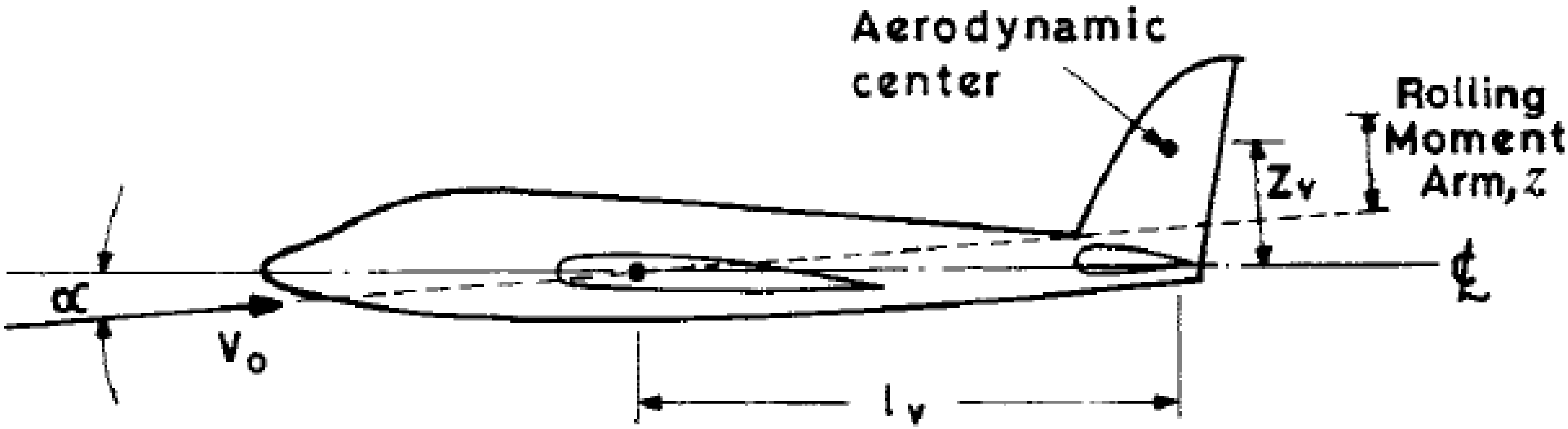
Fig B5

k



$\frac{b_V}{2r_1}$

Contribución Vertical C_{L_β}



$$(C_{l\beta})_V = -k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right) \eta_v \left(\frac{S_v}{S}\right) \left(\frac{z_v \cos\alpha - l_v \sin\alpha}{b}\right)$$

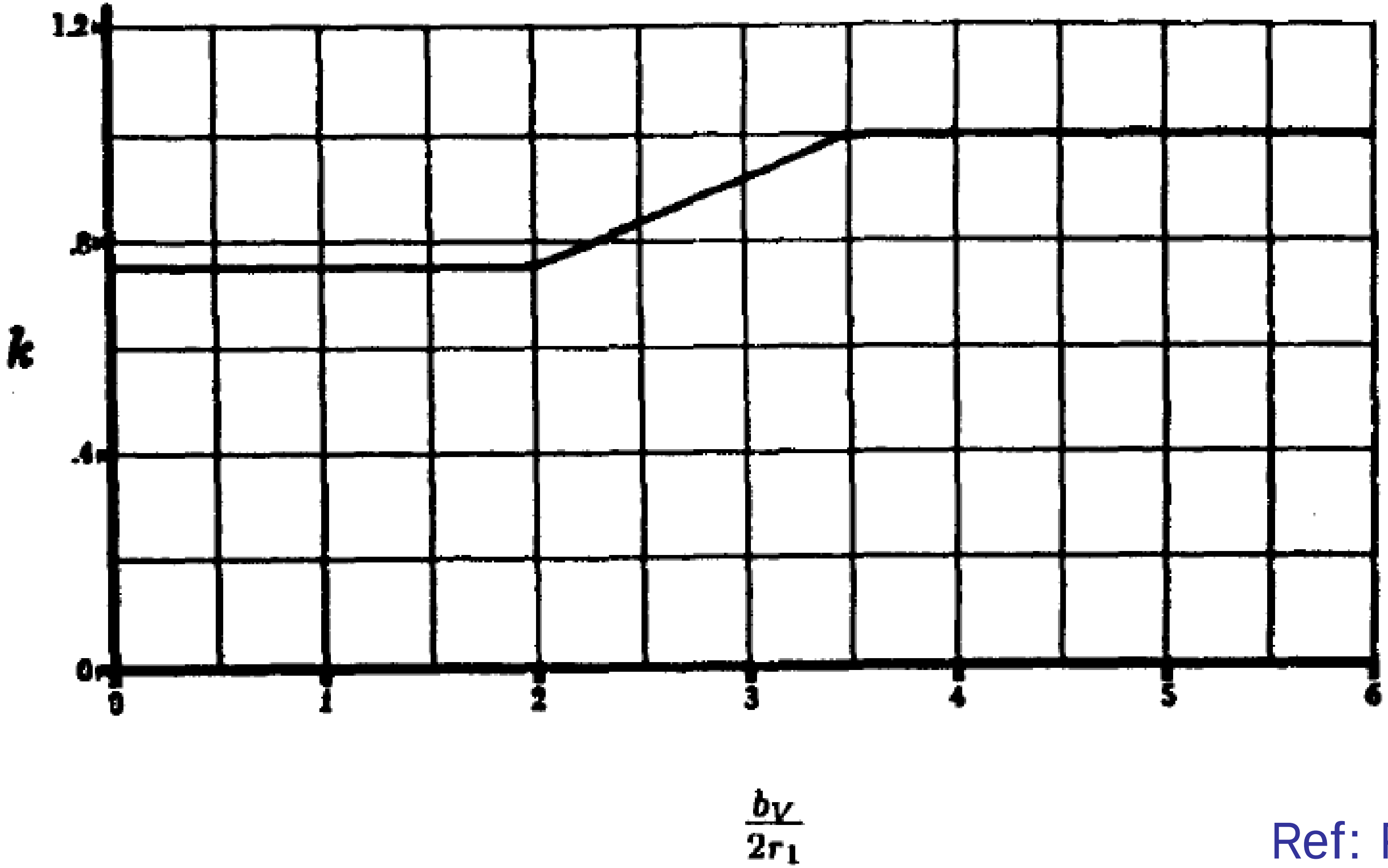
z is the rolling moment arm,

l_v is distance between the center of gravity and tail aerodynamic center measured along the airplane reference line,

z_v is the distance of the vertical tail aerodynamic center measured normal to the airplane centerline

Ref: Pamadi

Fig B5



Ref: Pamadi

Fig. 3.75 Empirical parameter k as a function of $b_V/2r_1$ (Ref. 1).

b_v vertical tail span measured up to the fuselage centerline

r_1 es average radius of the fuselage sections underneath the vertical tail

Contribución Canard – Horizontal - Vtail C_{L_β}

Método I

Contribución horizontal & V-tail (misma ecuación, utilizar proyección horizontal V-tail)

$$C_{l_{\beta_h}} = C_{l_{\beta_{hf}}} \frac{S_h b_h}{S_w b_w} \Rightarrow C_{l_{\beta_{hf}}} \Rightarrow$$ Contribución de estabilizador horizontal al efecto de diedro

Contribución canard

$$C_{l_{\beta_c}} = C_{l_{\beta_{cf}}} \frac{S_c b_c}{S_w b_w} \Rightarrow C_{l_{\beta_{cf}}} \Rightarrow$$ Contribución del canard al efecto de diedro

$C_{l_{\beta_{hf}}}$, $C_{l_{\beta_{cf}}}$ $\Rightarrow (C_{l\beta})_{W(B)} \Rightarrow$

$$(C_{l\beta})_{W(B)} = C_L\left[\left(\frac{C_{l\beta}}{C_L}\right)_{\Lambda_{c/2}} K_{M\Lambda}K_f + \left(\frac{C_{l\beta}}{C_L}\right)_A\right] + \Gamma\left[\frac{C_{l\beta}}{\Gamma}K_{M\Gamma} + \frac{\Delta C_{l\beta}}{\Gamma}\right] + (\Delta C_{l\beta})_{z_w}$$

emplear ecuaciones para contribución ala

S_h superficie estabilizador horizontal
S_c superficie canard
S_w superficie alar
b_h envergadura horizontal
b_c envergadura canard
b_w envergadura ala

Ref: Roskam

Contribución Ala (diedro) C_{N_β}

Wing contribution.

$$(C_{n\beta})_W = (C_{n\beta})_{\Gamma,W} + (C_{n\beta})_{\Lambda,W}$$

Effect of wing dihedral.

Contribución diedro

$$(C_{n\beta})_{\Gamma,W} = -\frac{2\Gamma}{Sb}\int_0^{b/2}(C_L - C_{D\alpha,l})c(y)y\,dy$$

For a rectangular wing with a constant chord c, →Asumir aproximación

$$(C_{n\beta})_{\Gamma,W} = -\frac{\Gamma(C_L - C_{D\alpha,l})}{4}$$

Diedro en radianes

$C_{D\alpha,l}$ →increase in section drag coefficient (2D) per unit increase in angle of attack

Usually for steady level flight conditions, $C_L > C_{D\alpha,l}$ desestabilizante

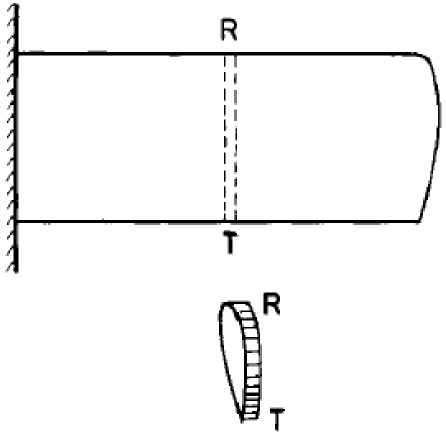
Strip Theory→ignorando los efectos de la resistencia inducida, el error aumenta a medida que AR se incrementa

$$(C_{n\beta})_{\Gamma,W} = -0.075\,\Gamma C_L/\mathrm{rad}$$

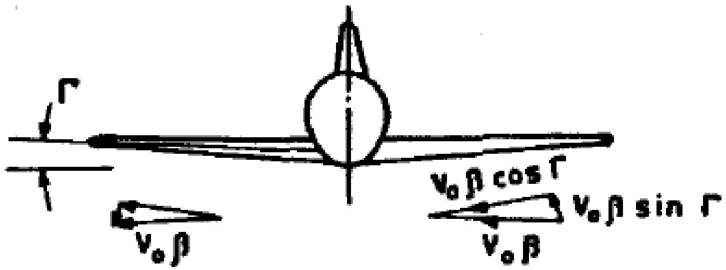
dihedral angle Γ is in radians

Ref: Pamadi

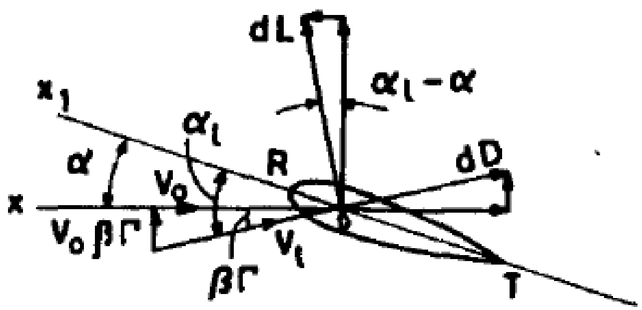
Lateral-Directional C_{N_β}

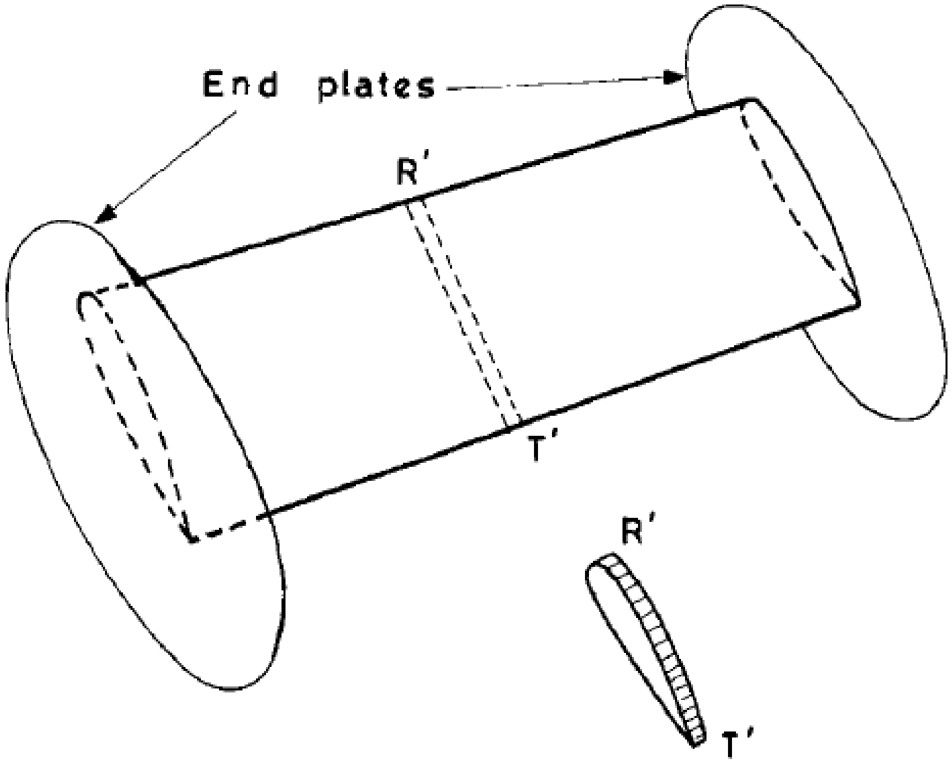


a) Strip *RT* on a finite wing



b)





b) Equivalent strip *R'T'* on a two-dimensional wing

Concept of strip theory (Pamadi)

Contribución Ala (flecha) C_{N_β}

Método I

Wing contribution.

$$(C_{n\beta})_W = (C_{n\beta})_{\Gamma,W} + (C_{n\beta})_{\Lambda,W}$$

Effect of sweep.

$$(C_{n\beta})_{\Lambda,W} = \left(\frac{2\sin\Lambda}{Sb}\right)\int_0^{\frac{b\sec\Lambda}{2}} (C_{L,l}\alpha + 2C_{DO,l}\cos\Lambda + C_{D\alpha,l}\alpha)c(y_h)y_h\,\mathrm{d}y_h$$

1) the sweep-back has a stabilizing effect on static directional stability and
2) the stabilizing effect increases with angle of attack

$$\frac{(C_{n\beta})_{\Lambda,W}}{C_L^2} = \frac{1}{4\pi A} - \frac{\tan\Lambda_{c/4}}{\pi A(A+4\cos\Lambda_{c/4})}\times\left(\cos\Lambda_{c/4} - \frac{A}{2} - \frac{A^2}{8\cos\Lambda_{c/4}} - 6\bar{x}_a\frac{\sin\Lambda_{c/4}}{A}\right) \qquad \frac{1}{rad}$$

resolver ➡ $\frac{(C_{n\beta})_{\Lambda,W}}{C_L^2}$ X C_L^2 ➡ $(C_{n\beta})_{\Lambda,W}$

$\Lambda_{c/4}$ is the wing quarter chord sweep, ➡ $\tan\Lambda_{c/4} = \tan\Lambda_{LE} - \left(\frac{c_r - c_t}{2b}\right)$

A is the wing aspect ratio (theoretical)

$\bar{x}_a$ is the distance between the center of gravity and the wing aerodynamic center in terms of mean aerodynamic chord of the wing.

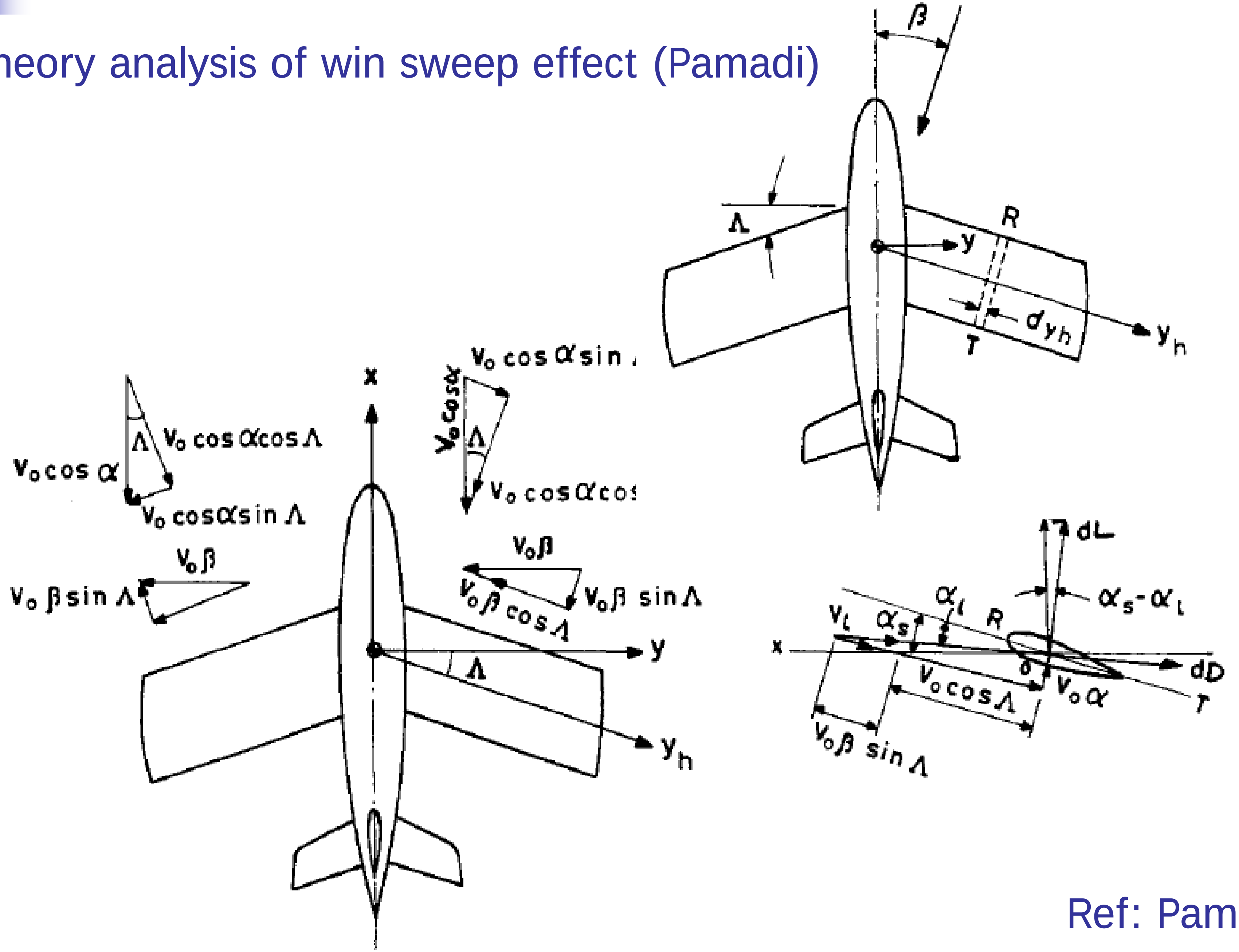
$\bar{x}_a > 0$ if the center of gravity is aft of the wing aerodynamic center.

c_r root chord
c_t tip chord
b wing span
Λ_{LE} leading edge sweept

Ref: Pamadi

Lateral-Directional C_{N_β}

Strip theory analysis of win sweep effect (Pamadi)



Ref: Pamadi

Contribución Vertical - I

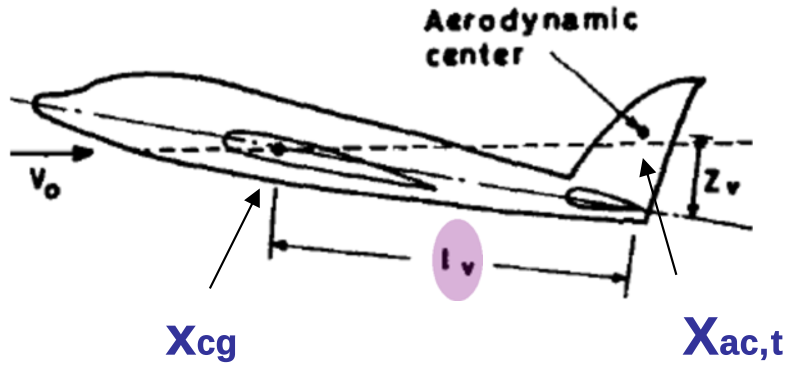
C_{N_β}

Contribución vertical

Tail contribution.

Método I

$$(C_{n\beta,V})_{\text{fix}} = k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v \bar{V}_2$$



$$a_v = a_w = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{k^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{\beta^2}\right) + 4}}$$

lift-curve slope of the vertical tail

$$\bar{V}_2 = \frac{S_v l_v}{Sb}$$

$$\left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v = 0.724 + \frac{3.06 S_v/S}{1 + \cos\Lambda_{c/4}} + \frac{0.4 z_w}{d_{f,\max}} + 0.009A$$

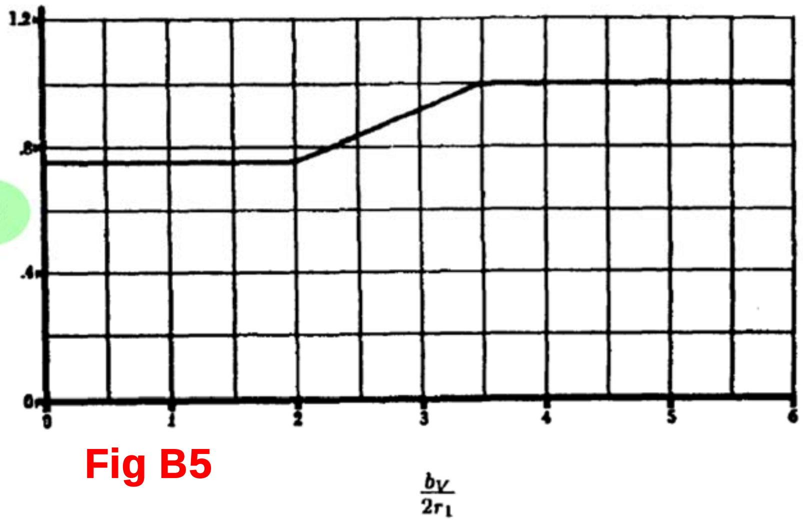


Fig B5

$d_{f,\max}$ is the maximum fuselage depth,

A the theoretical wing aspect ratio,

$\Lambda_{c/4}$ is the theoretical wing quarter chord sweep,

a_v lift-curve slope of the vertical tail →Se puede obtener mediante estudio aerodinámica

z_w is the vertical distance (measured parallel to the z axis) from wing root quarter chord point to the fuselage centerline, positive if the wing is below the fuselage centerline.

Ref: Pamadi

Contribución Vertical - II C_{N_β}

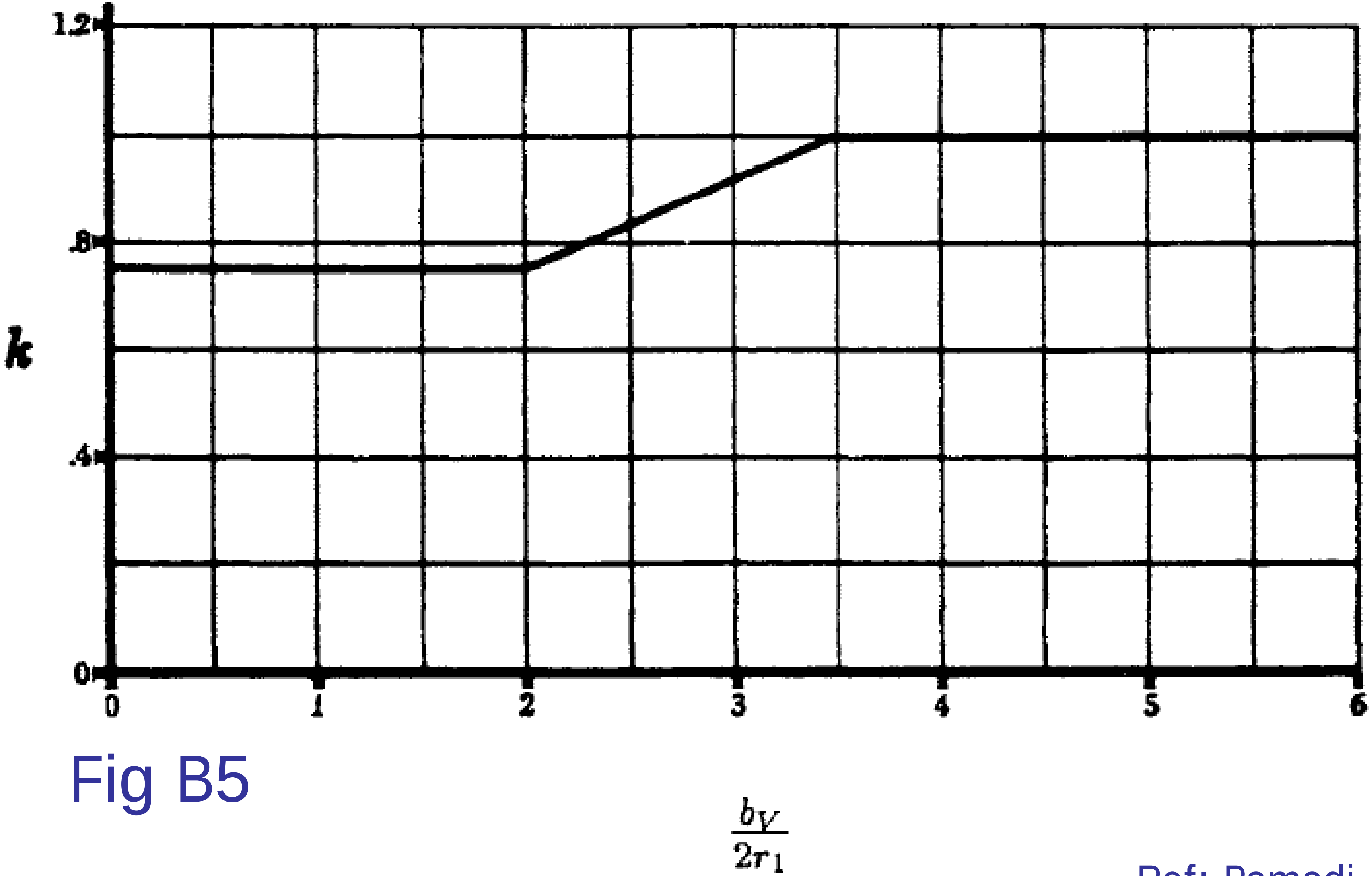


Fig B5

Fig. 3.75 Empirical parameter k as a function of $b_V/2r_1$ (Ref. 1).

Ref: Pamadi

Contribución Vertical - III

C_{N_β}

Pendiente de sustentación se puede obtener

Fig A19

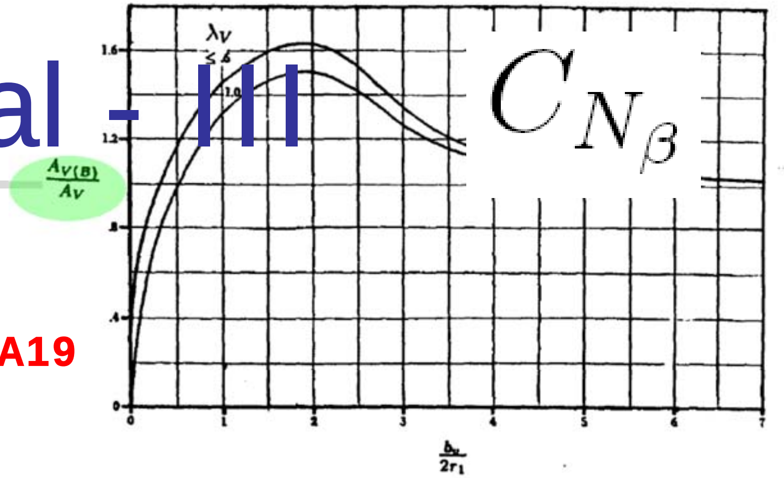


Fig. 3.77 Empirical parameter $A_{V(B)}/A_V$ as a function of $b_V/2r_1$ (Ref.1).

También mediante métodos teóricos

a_v lift-curve slope of the vertical tail

$$a_w = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{k^2}\left(1 + \frac{\tan^2 \Lambda_{c/2}}{\beta^2}\right) + 4}}$$

Fig A20

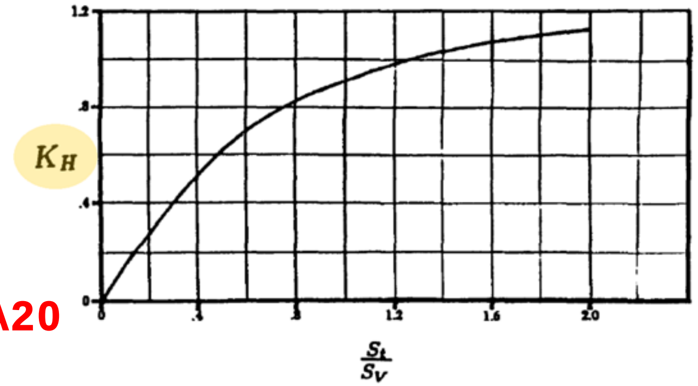


Fig. 3.79 Empirical parameter K_H as a function S_t/S_V (Ref. 1).

Método válido para cualquier superficie aerodinámica

Se ha de utilizar el effective Aspect Ratio $A_{v,eff}$

$$A_{v,\text{eff}} = \left(\frac{A_{v(B)}}{A_v} A_v\right)\left[1 + K_H\left(\frac{A_{v(HB)}}{A_{v(B)}} - 1\right)\right]$$

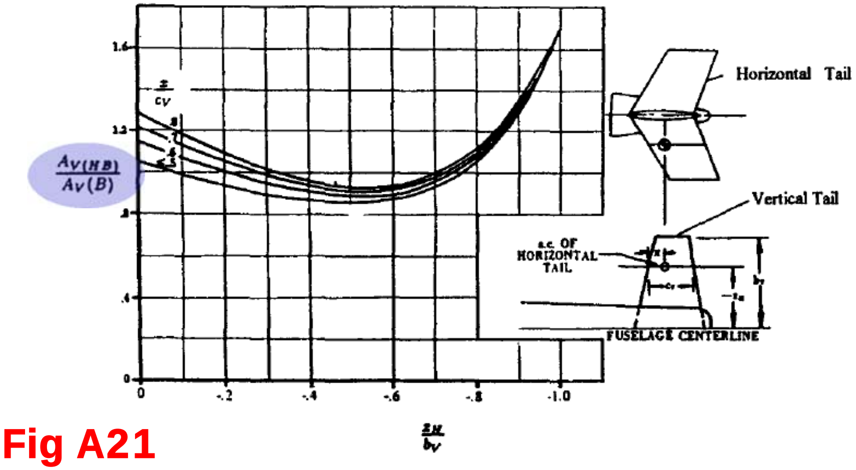


Fig A21

Fig. 3.78 Empirical parameter $A_{V(HB)}/A_{V(B)}$ as a function of z_H/b_V (Ref. 1).

Ref: Pamadi

Contribución Vertical - IV C_{N_β}

$$A_{v,\text{eff}} = \left(\frac{A_{v(B)}}{A_v} A_v\right)\left[1 + K_H\left(\frac{A_{v(HB)}}{A_{v(B)}} - 1\right)\right]$$

$A_{v(B)}/A_v$ is the ratio of the vertical tail aspect ratio in the presence of the body to that of the isolated vertical tail

b_v, the vertical tail span measured up to the fuselage centerline;

$2r_1$, fuselage average depth in the region of vertical tail;

λ_v, vertical tail taper ratio based on vertical tail surface measured up to the fuselage centerline.

A_v is the geometrical aspect ratio of the vertical tail with span and area measured up to the fuselage centerline

$A_{v(HB)}/A_{v(B)}$ is the ratio of vertical tail aspect ratio in the presence of the horizontal tail and body to that of the vertical tail in the presence of the body alone

K_H is a factor accounting for the relative size of the horizontal and vertical tails

Ref: Pamadi

Contribución Vertical - V C_{N_β}

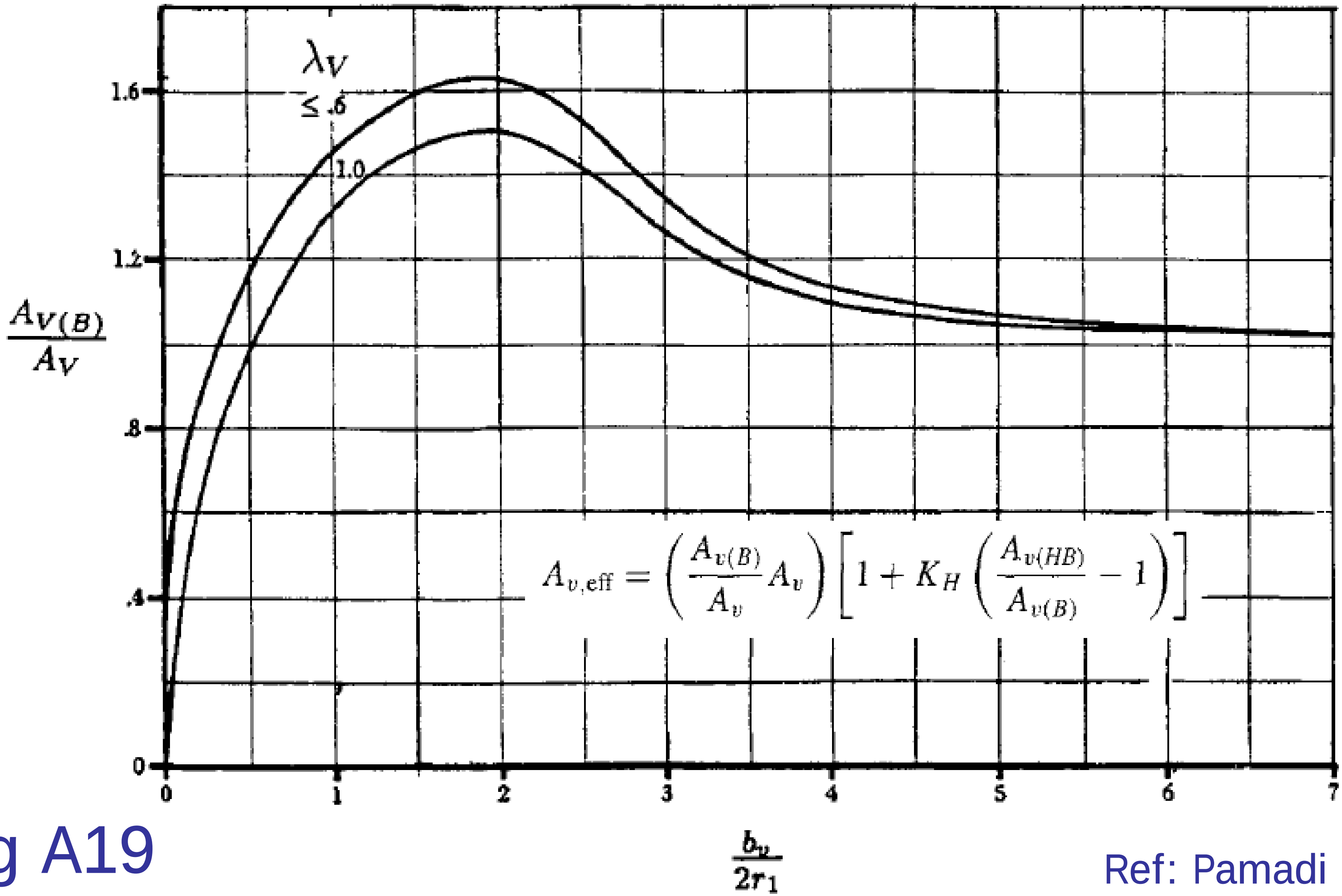


Fig A19

Ref: Pamadi

Fig. 3.77 Empirical parameter $A_{V(B)}/A_V$ as a function of $b_V/2r_1$ (Ref. 1).

Contribución Vertical - VI

C_{N_β}

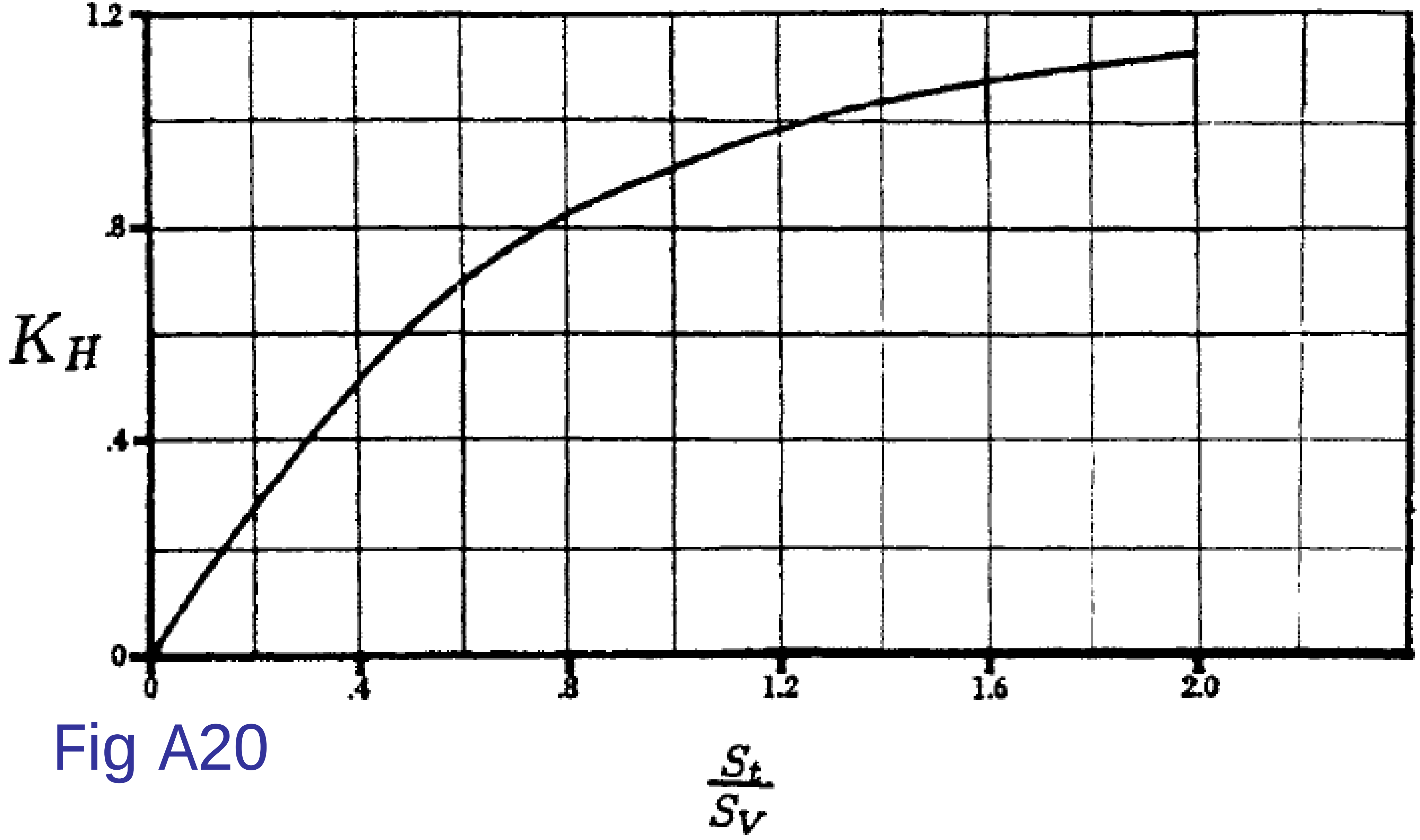


Fig A20

Fig. 3.79 Empirical parameter K_H as a function S_t/S_V (Ref. 1).

Ref: Pamadi

Contribución Vertical - VII C_{N_β}

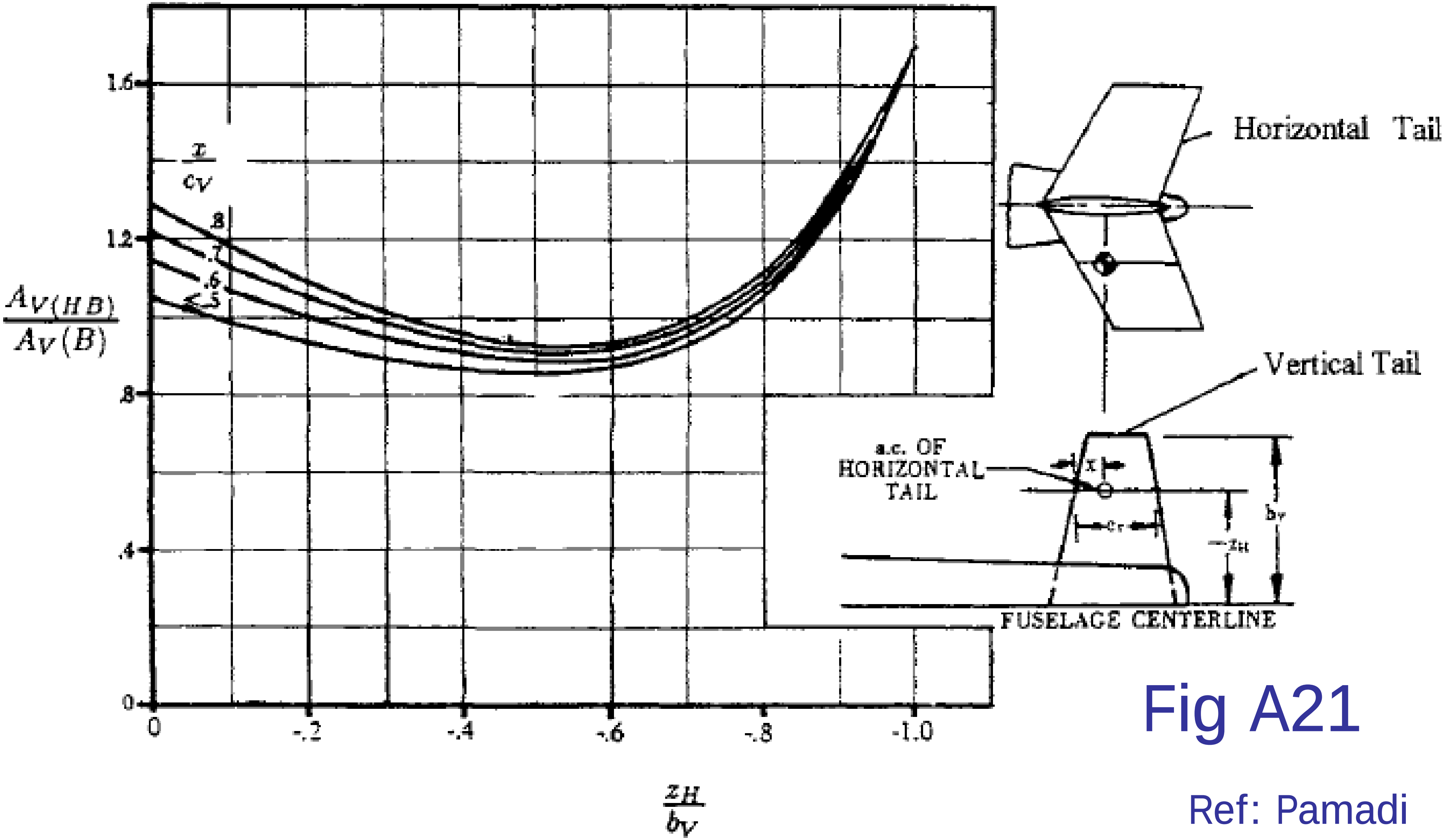


Fig A21

Ref: Pamadi

Fig. 3.78 Empirical parameter $A_{V(HB)}/A_{V(B)}$ as a function of z_H/b_V (Ref. 1).

Contribución Vertical - VIII C_{N_β}

Método II

$$C_{n_{\beta_v}} = -C_{y_{\beta_v}} \frac{\left(X_{ac_v} - X_{cg}\right)\cos\alpha + \left(Z_{ac_v} - Z_{cg}\right)\sin\alpha}{b_w}$$

Ref: DARCorp

where:

$C_{y_{\beta_{vf}}}$	is the ventral fin contribution to the sideforce-coefficient-due-to-sideslip derivative.
$Z_{ac_{vf}}$	is the Z-coordinate of the vertical tail aerodynamic center.
Z_{cg}	is the Z-coordinate of the airplane center of gravity.
$X_{ac_{vf}}$	is the X-coordinate of the vertical tail aerodynamic center.
X_{cg}	is the X-coordinate of the airplane center of gravity.
α	is the airplane angle of attack.
b_w	is the wing span.

Contribución Fuselaje - I

C_{N_β}

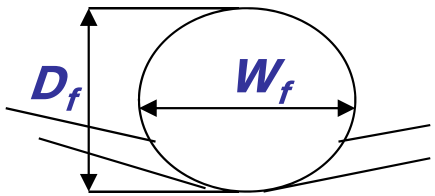
Contribución Fuselaje

Fig B3

Método I

$$C_{n_{\beta_{fus}}} = -1.3\,\frac{\text{volume}}{S_w b}\left(\frac{D_f}{W_f}\right)$$

Fuselaje (góndola) profundidad y anchura



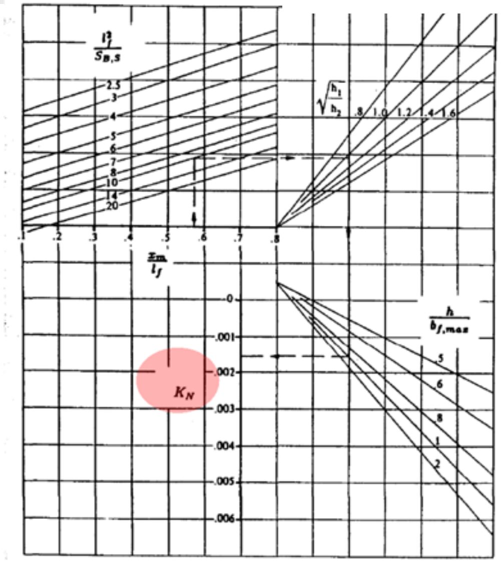


Fig. 3.73 Empirical factor K_N related to $(C_{n\beta})_{B(W)}$ (Ref. 1).

Fuselage contribution.

Método II

$$(C_{n\beta})_{B(W)} = -K_N K_{RI}\left(\frac{S_{B.S}}{S}\right)\left(\frac{l_f}{b}\right)/\text{deg}$$

Hay que convertir a 1/rad

$$(C_{n\beta})_{B(W)}\ x\ \frac{180}{\pi} = \frac{1}{rad}$$

K_N is an empirical wing–body interference factor

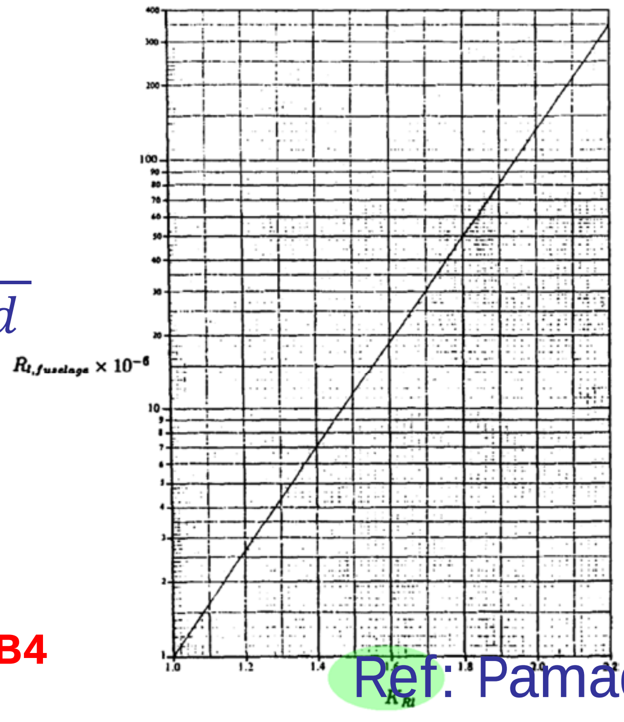
K_{RI} is an empirical factor

l_f is the length of the fuselage.

$S_{B,S}$ is the projected side area of the fuselage.

S is the reference wing area,

Fig B4



Ref: Pamadi

Fig. 3.74 Variation of K_{RI} with fuselage Reynolds number.[1]

Fig B3

Ref: Pamadi

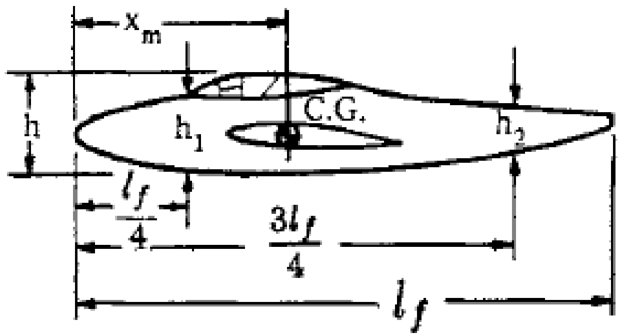
1) Identificar $\frac{x_m}{l_f}$

2) Para un $\frac{x_m}{l_f}$ determinar $\frac{l_f{}^2}{S_{B,S}}$ más próximo

3) Para un $\frac{l_f{}^2}{S_{B,S}}$ determinar $\sqrt{\frac{h_1}{h_2}}$ más próximo

4) Para un $\sqrt{\frac{h_1}{h_2}}$ determinar $\frac{h}{b_{f,max}}$ más próximo

5) Para un $\frac{h}{b_{f,max}}$ determinar K_N



$S_{B,S}$: **Body Side Area**

$b_{f,max}$: **Maximum Body Width**

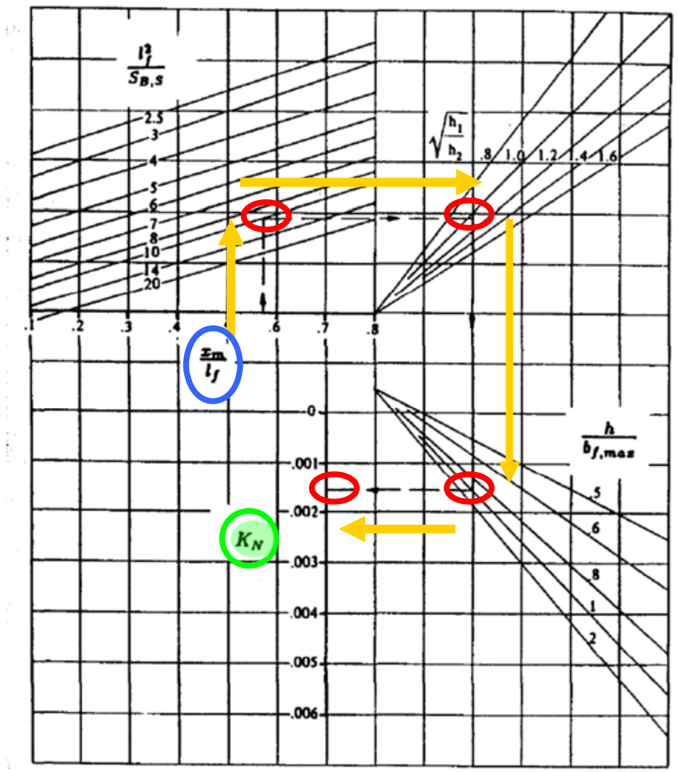
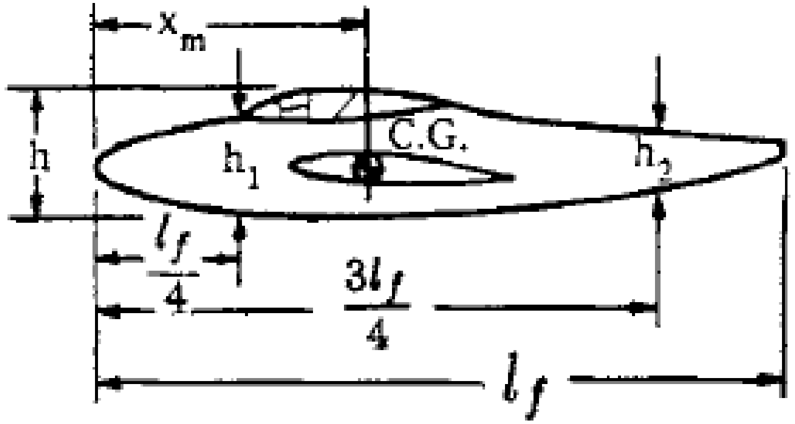


Fig. 3.73 Empirical factor K_N related to $(C_{n\beta})_{B(W)}$ (Ref. 1).

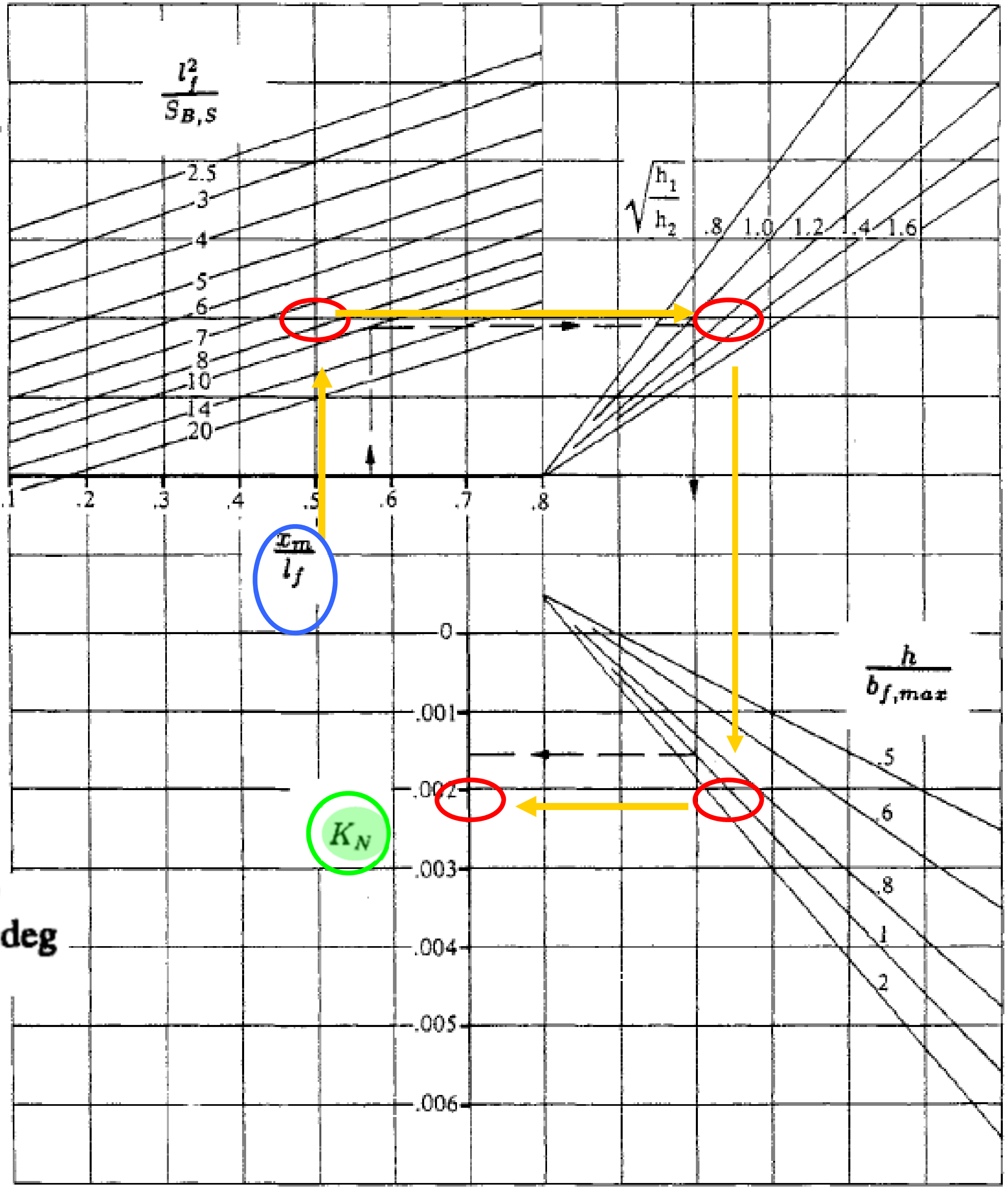
Fig B3



$S_{B,S}$: **Body Side Area**

$b_{f,max}$: **Maximum Body Width**

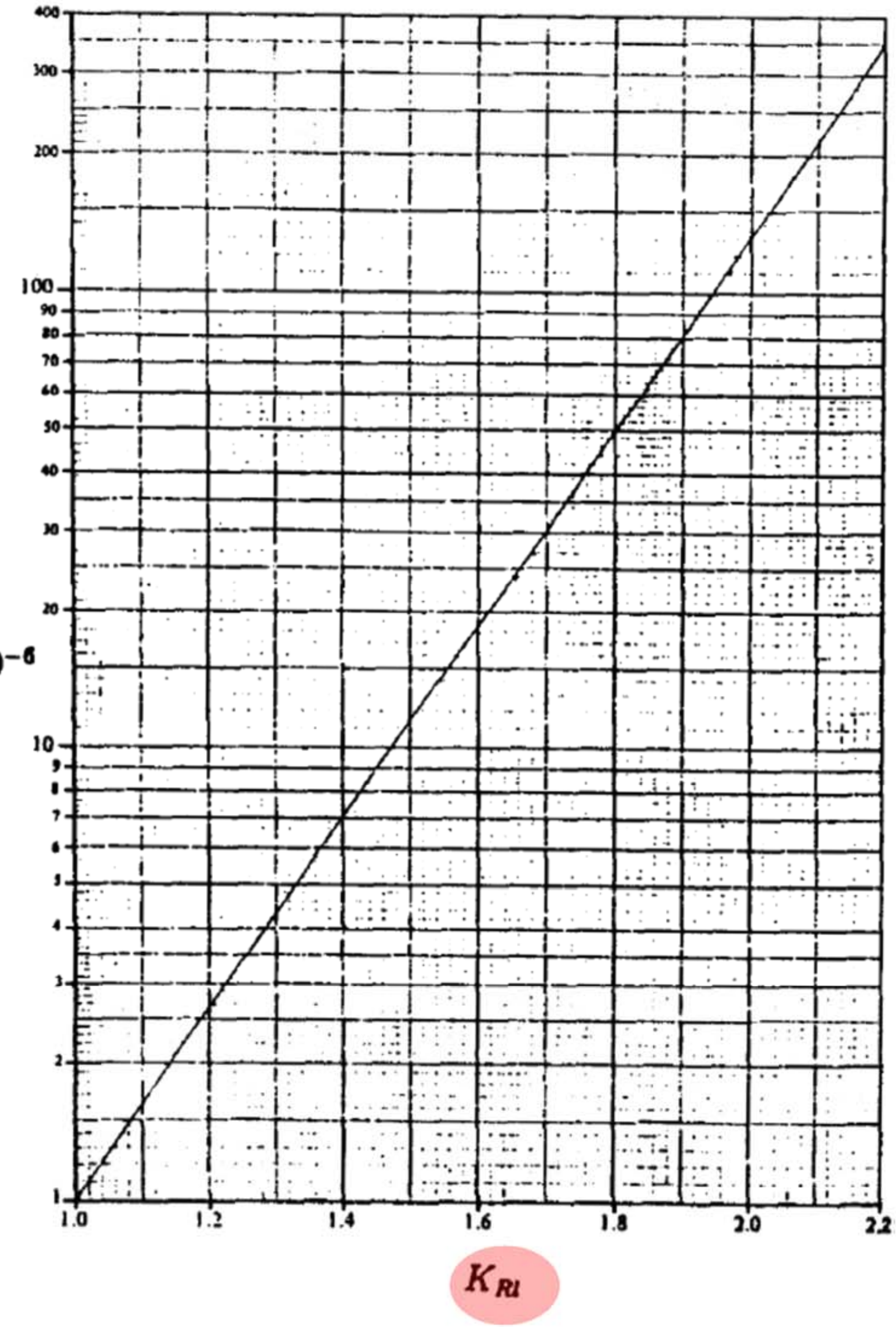
$$(C_{n\beta})_{B(W)} = -K_N K_{RI} \left(\frac{S_{B,S}}{S}\right)\left(\frac{l_f}{b}\right) / \text{deg}$$



Ref: Pamadi

Fig B4

Num Reynolds del fuselaje



$$(C_{n\beta})_{B(W)} = -K_N K_{Rl}\left(\frac{S_{B.S}}{S}\right)\left(\frac{l_f}{b}\right) / \text{deg}$$

Fig. 3.74 Variation of K_{Rl} with fuselage Reynolds number.[1]

Ref: Pamadi

Contribución Fuselaje - II C_{N_β}

The value of K_β, an empirical constant, can be obtained from the following graph as a function of fineness ratio and c.g. location. The distance from the nose to the c.g. is d; S_s is the body side area. The other variables are shown below.

A value of $(C_{n_\beta})_{fus}$ can be obtained from Perkins and Hage by use of

Método III

$$(C_{n_\beta})_{fus} = \frac{-.96\ K_\beta}{57.3}\left(\frac{S_s}{S_w}\right)\left(\frac{\ell_b}{b_w}\right)\left(\frac{h_1}{h_2}\right)^{1/2}\left(\frac{w_2}{w_1}\right)^{1/3}$$

B6

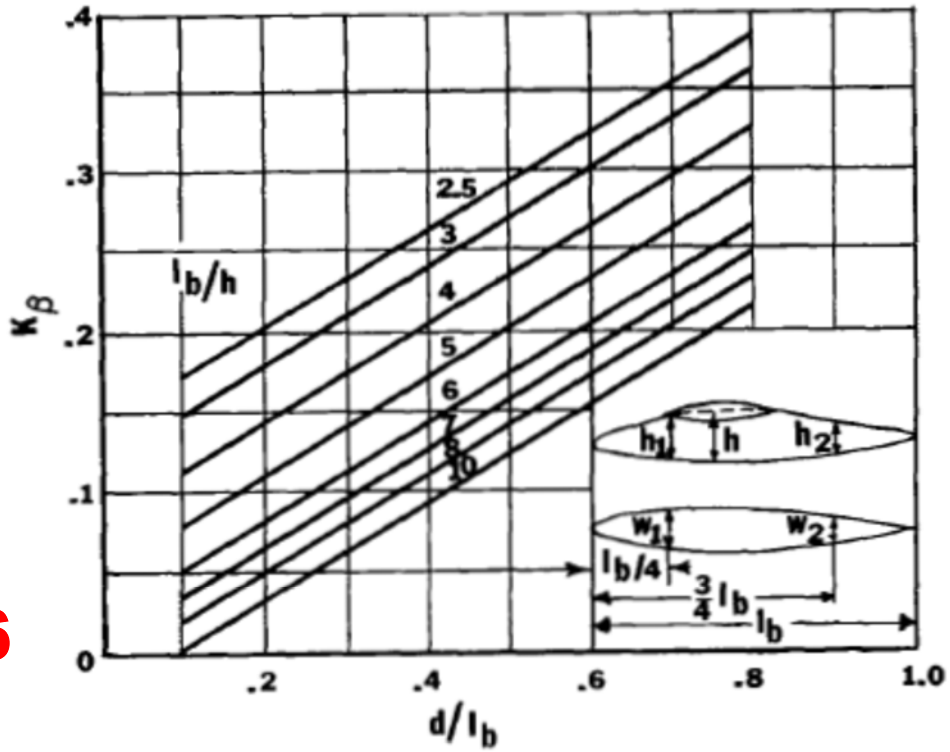


Figure 27. Empirical constant K_β as a function of fineness ratio and c.g. location.

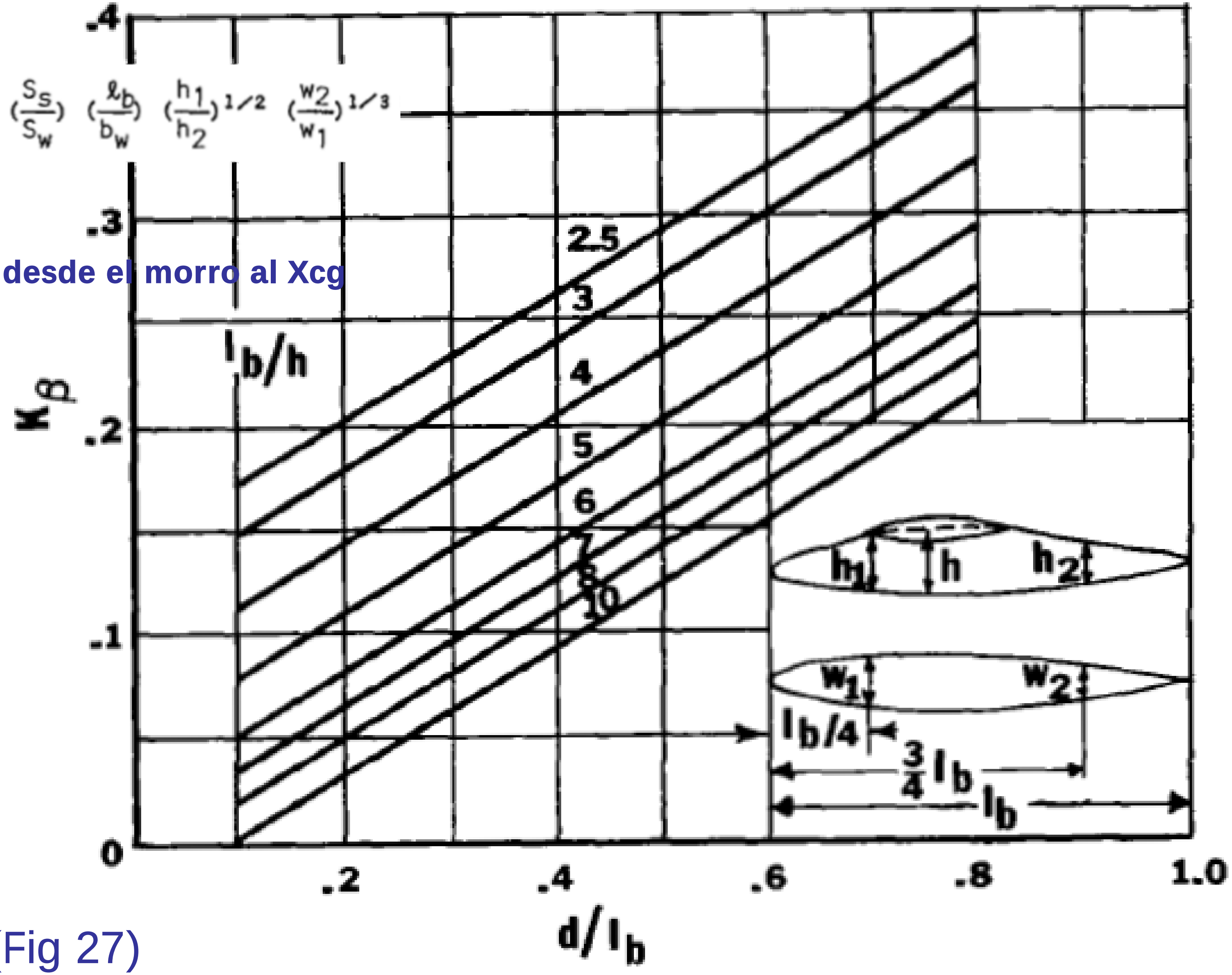
Ref: Smetana

Fig B6

C_{N_β}

$$(C_{n_\beta})_{fus} = \frac{-.96\, K_\beta}{57.3}\left(\frac{S_s}{S_w}\right)\left(\frac{\ell_b}{b_w}\right)\left(\frac{h_1}{h_2}\right)^{1/2}\left(\frac{w_2}{w_1}\right)^{1/3}$$

d es la distancia desde el morro al Xcg



Ref: Smetana (Fig 27)

Lateral-Directional $C_{Y_{T_\beta}}$

For a propeller-driven airplane:
The airplane thrust sideforce-coefficient-due-to-sideslip-derivative is given by:

$$C_{y_{T_\beta}} = -\frac{\frac{\pi}{4} N_{prop} D_{prop}^2 \left(\frac{dC_N}{d\alpha} \right)_{prop}}{S_w}$$

where:

N_{prop}	is the number of propellers per airplane.
l_{prop}	is the moment arm of the propeller normal force to the airplane center of gravity.
D_{prop}	is the propeller diameter.
$\left(\frac{dC_N}{d\alpha}\right)_{prop}$	is the change in propeller normal force coefficient with angle of attack.
S_w	is the wing area.
b_w	is the wing span.

Para aviones jet ⇨ Aproximación $C_{Y_{T_\beta}} \approx 0$

Para aviones con hélice ⇨ Muy compleja estimación ⇨ Aproximación $C_{Y_{T_\beta}} \approx 0$

Propulsive Derivatives $C_{Y_{T_\beta}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$

where:

K_N	is the first intermediate calculation parameter.
$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$	is the second intermediate calculation parameter.

The first intermediate calculation parameter is given by

$$K_N = 262\left(\frac{w}{R}\right)_{0.3R_{prop}} + 262\left(\frac{w}{R}\right)_{0.6R_{prop}} + 135\left(\frac{w}{R}\right)_{0.9R_{prop}}$$

Geometría de la hélice

where:

$\left(\frac{w}{R}\right)_{0.3R_{prop}}$	is the propeller blade width-to-radius ratio at 30% radius.
$\left(\frac{w}{R}\right)_{0.6R_{prop}}$	is the propeller blade width-to-radius ratio at 60% radius.
$\left(\frac{w}{R}\right)_{0.9R_{prop}}$	is the propeller blade width-to-radius ratio at 90% radius.

The propeller blade radius

$$R_{prop} = \frac{D_{prop}}{2}$$

D_{prop} diameter of prop

Propulsive Derivatives $C_{Y_{T_\beta}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$

The second intermediate calculation parameter is obtained from Figure 8.130 in Airplane Design Part VI and is a function of the number of propeller blades and the nominal propeller blade angle at 75% radius.

$$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7} \Rightarrow$$

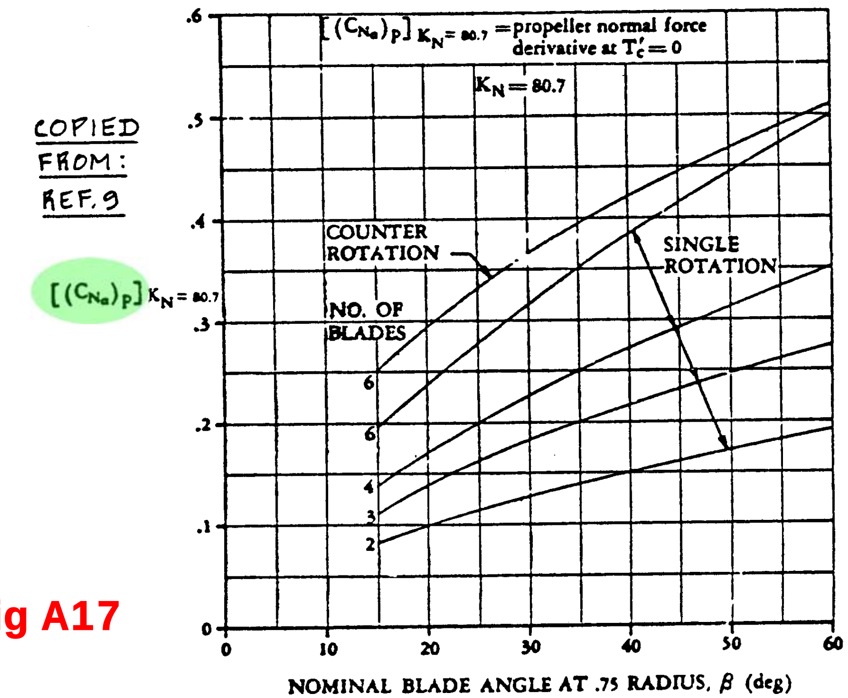


Fig A17

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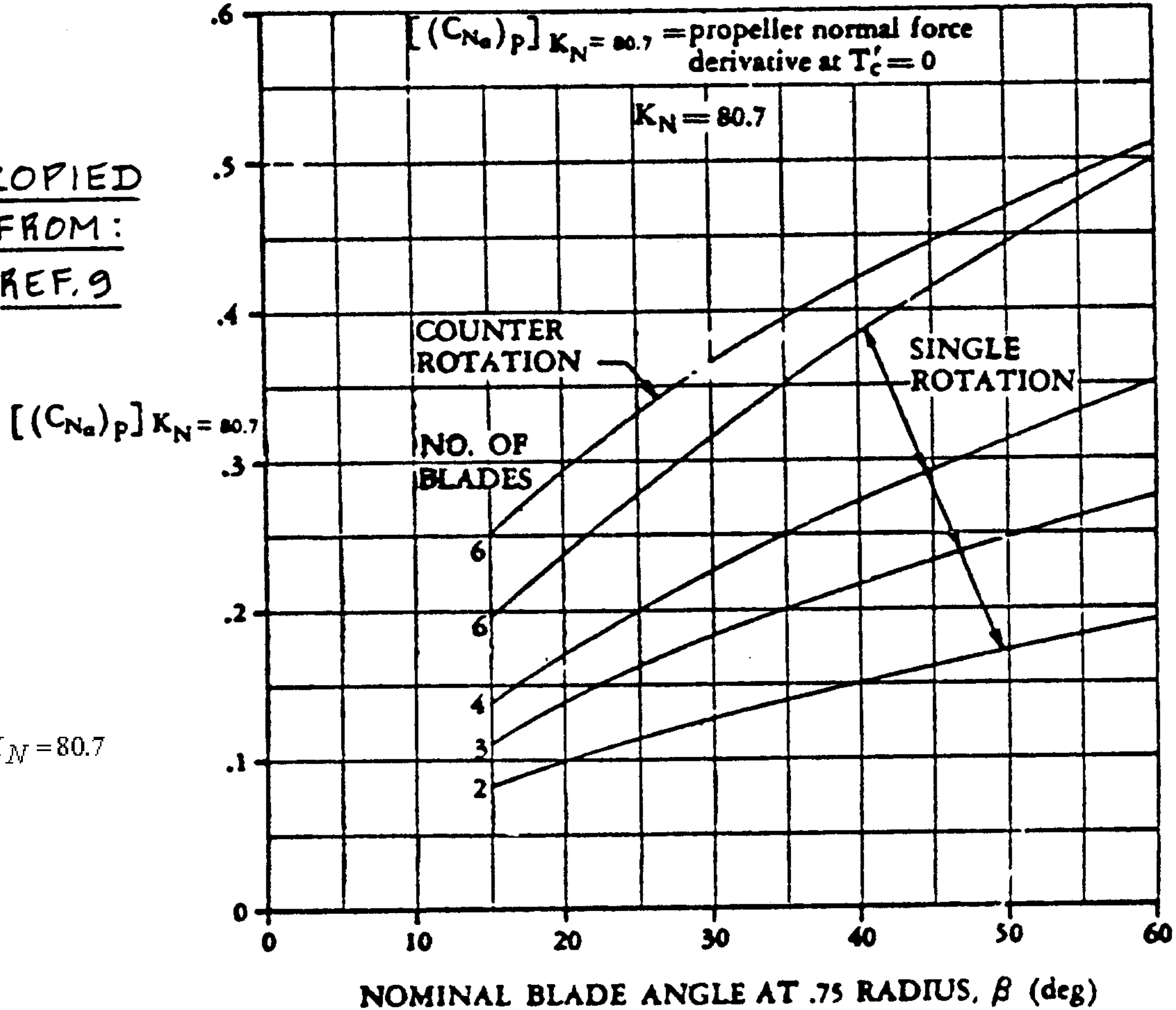


$$\left[\left(C_{N\alpha}\right)_p\right]_{K_N=80.7}$$

Lateral-Directional $C_{N_{T_\beta}}$

For a propeller-driven airplane:
The airplane thrust yawing-moment-coefficient-due-to-sideslip-derivative is given by

$$C_{n_{T_\beta}} = -\frac{\frac{\pi}{4} N_{prop} l_{prop} D_{prop}^2 \left(\frac{dC_N}{d\alpha}\right)_{prop}}{S_w b_w}$$

where:

N_{prop}	is the number of propellers per airplane.
l_{prop}	is the moment arm of the propeller normal force to the airplane center of gravity.
D_{prop}	is the propeller diameter.
$\left(\frac{dC_N}{d\alpha}\right)_{prop}$	is the change in propeller normal force coefficient with angle of attack.
S_w	is the wing area.
b_w	is the wing span.

Para aviones jet ⇒ Aproximación $C_{N_{T_\beta}} \approx 0$

Para aviones con hélice ⇒ Muy compleja estimación ⇒ Aproximación $C_{N_{T_\beta}} \approx 0$

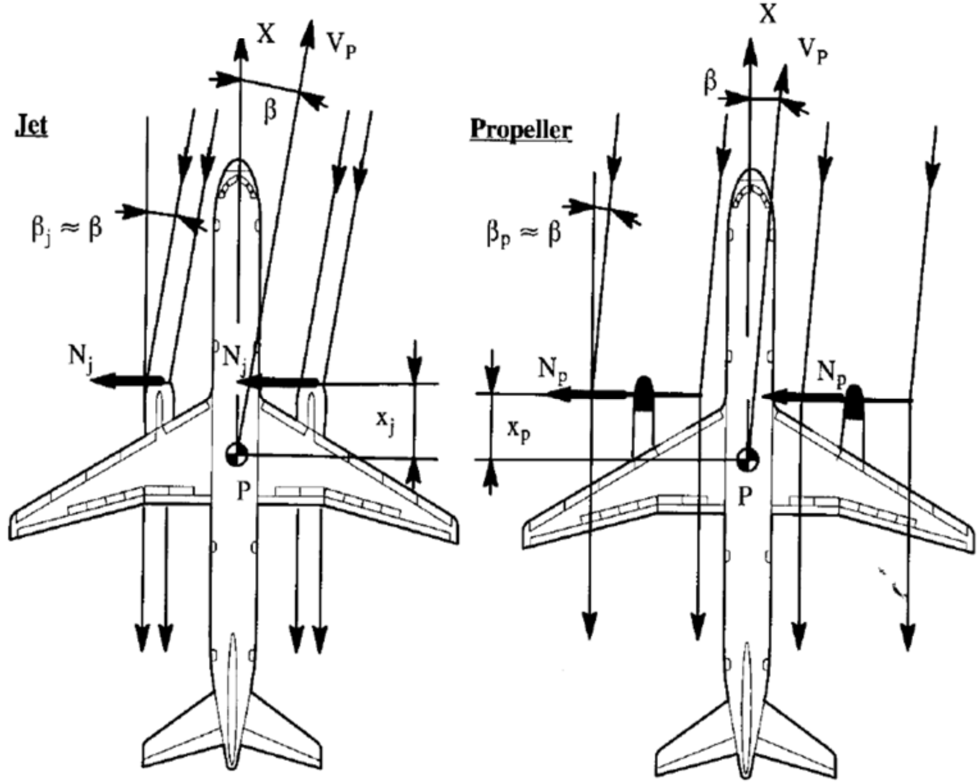
Lateral-Directional $C_{N_{T_\beta}}$

For a propeller-driven airplane:
The moment arm of the propeller normal force to the airplane center of gravity is given

$$l_{prop} = \left(X_{cg} - X_{prop}\right)\cos\psi_T - \left(Y_{cg} - Y_{prop}\right)\sin\psi_T$$

where:

X_{cg}	is the X-coordinate of airplane center of gravity.
X_{prop}	is the X-coordinate of the propeller.
Y_{prop}	is the Y-coordinate of the propeller.
Y_{cg}	is the Y-coordinate of airplane center of gravity.
Φ_T	is the thrust line inclination angle.



Para aviones jet ⇨ Aproximación $C_{N_{T_\beta}} \approx 0$

Para aviones con hélice ⇨ Muy compleja estimación ⇨ Aproximación $C_{N_{T_\beta}} \approx 0$

Propulsive Derivatives $C_{N_{T_\beta}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$

where:

K_N	is the first intermediate calculation parameter.
$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$	is the second intermediate calculation parameter.

The first intermediate calculation parameter is given by

$$K_N = 262\left(\frac{w}{R}\right)_{0.3R_{prop}} + 262\left(\frac{w}{R}\right)_{0.6R_{prop}} + 135\left(\frac{w}{R}\right)_{0.9R_{prop}}$$

Geometría de la hélice

where:

$\left(\frac{w}{R}\right)_{0.3R_{prop}}$	is the propeller blade width-to-radius ratio at 30% radius.
$\left(\frac{w}{R}\right)_{0.6R_{prop}}$	is the propeller blade width-to-radius ratio at 60% radius.
$\left(\frac{w}{R}\right)_{0.9R_{prop}}$	is the propeller blade width-to-radius ratio at 90% radius.

The propeller blade radius

$$R_{prop} = \frac{D_{prop}}{2}$$

D_{prop} diameter of prop

Propulsive Derivatives $C_{N_{T_\beta}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$

The second intermediate calculation parameter is obtained from Figure 8.130 in Airplane Design Part VI and is a function of the number of propeller blades and the nominal propeller blade angle at 75% radius.

$$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$$

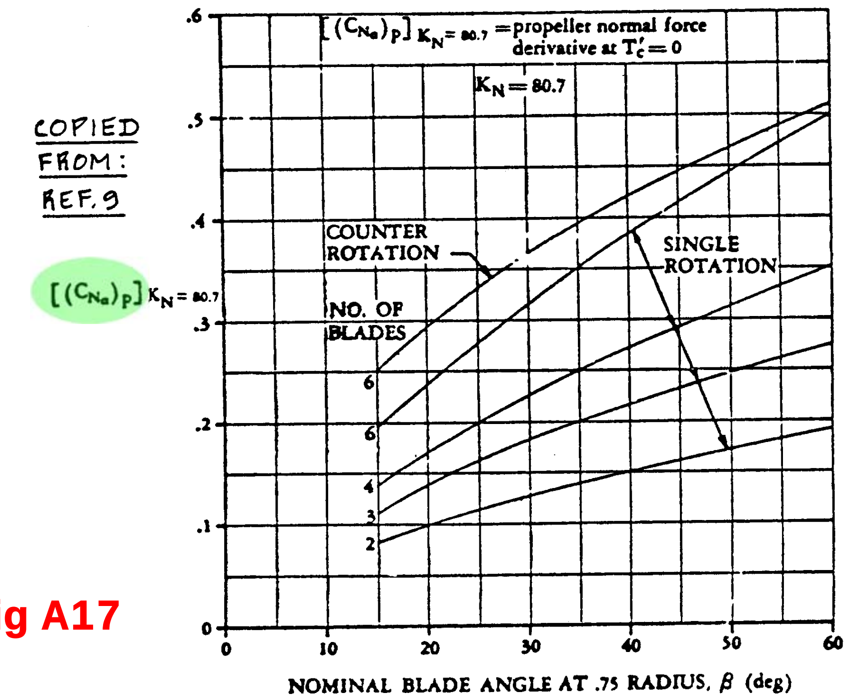


Fig A17

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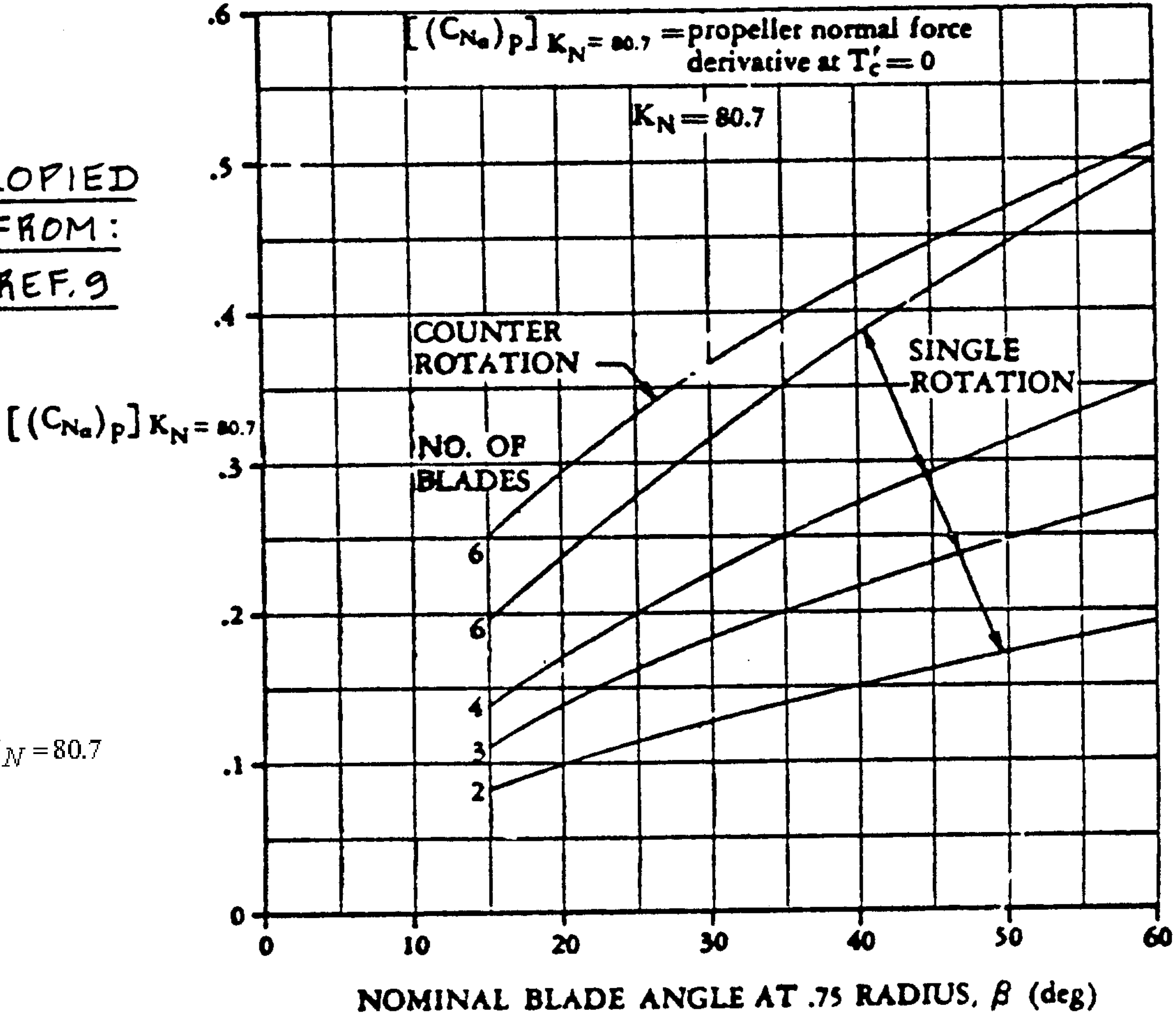


$$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$$

Lateral-Directional $C_{N_{T_\beta}}$

- Asumir que

$$C_{N_{T\beta}} \approx 0$$

Derivadas $C_{Y_p}, C_{L_p}, C_{N_p}$

Roll Rate Derivatives

Estimación Derivadas

- Contribución C_{Y_p}
  - Ala: flecha, diedro
  - Vertical
- Contribución C_{L_p}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje
  - Horizontal/Canard/V-tail
- Contribución C_{N_p}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje

Lateral-Directional C_{Y_p}

Método I → Sólo considera la contribución del ala

Ref: Pamadi

wing contribution $(C_{yp})_W$

$$K = \frac{1 - a_{w1}}{1 - a_{w2}} \qquad a_{w1} = \frac{(C_{L\alpha})_e}{\pi A e} \qquad a_{w2} = e a_{w1}$$

$$(C_{yp})_W = K \left(\frac{C_{yp}}{C_L} \right)_{C_L = 0, M} C_L + (\Delta C_{yp})_\Gamma / \text{rad}$$

$$\left(\frac{C_{yp}}{C_L} \right)_{C_L = 0, M} = \frac{(A + B \cos \Lambda_{c/4})(AB + \cos \Lambda_{c/4})}{(AB + 4 \cos \Lambda_{c/4})(A + \cos \Lambda_{c/4})} \left(\frac{C_{yp}}{C_L} \right)_{C_L = 0, M = 0}$$

$$B = \sqrt{1 - M^2 \cos^2 \Lambda_{c/4}}$$

$$A = A_e$$

A_e, the exposed wing aspect ratio.

$\left(\frac{C_{yp}}{C_L} \right)_{C_L = 0, M = 0}$ → Datos experimentales →

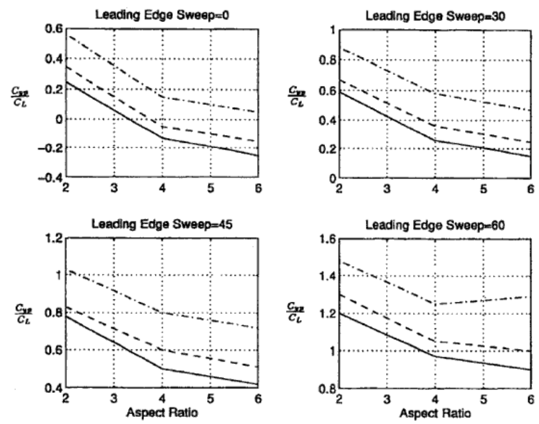
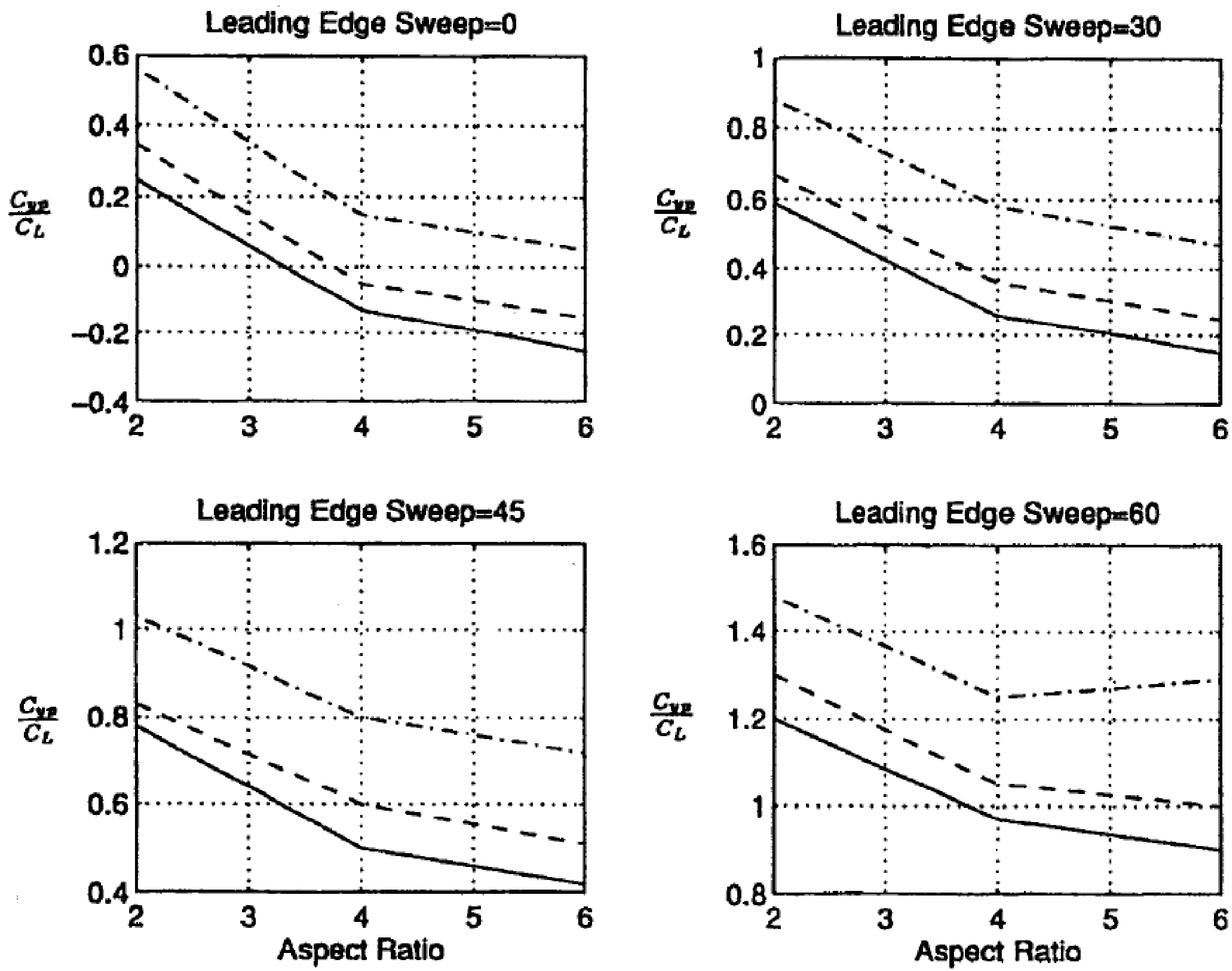


Fig. 4.24 The parameter $(C_{yp}/C_L)_{C_L=0,M=0}$ at supersonic speeds.[7]

Lateral-Directional C_{Y_p}

Método I
Ref: Pamadi



—— $\lambda = 0$
- - - - $\lambda = 0.5$
— · — · $\lambda = 1.0$

$$\left(\frac{C_{yp}}{C_L}\right)_{C_L=0, M=0}$$

Fig. 4.24 **The parameter $(C_{yp}/C_L)_{C_L=0,M=0}$ at supersonic speeds.**[7]

Lateral-Directional C_{Y_p}

Método I

Ref: Pamadi

wing contribution $(C_{yp})_W$

$$(C_{yp})_W = K\left(\frac{C_{yp}}{C_L}\right)_{C_L=0,M} C_L + (\Delta C_{yp})_\Gamma/\text{rad}$$

$$(\Delta C_{yp})_\Gamma = 3\sin\Gamma\left[1-\frac{4z}{b}\sin\Gamma\right](C_{lp})_{\Gamma=0,C_L=0}/\text{rad}$$

$$(C_{lp})_{\Gamma=0,C_L=0} = \left(\frac{\beta C_{lp}}{k}\right)_{C_L=0}\frac{k}{\beta}$$

$$z = z_v\cos\alpha - l_v\sin\alpha$$

Γ is the wing dihedral in deg

$$\beta = \sqrt{1-M^2},\qquad k = \frac{a_o}{2\pi}$$

a_o is the sectional or two-dimensional lift-curve slope of the wing at low subsonic speeds

$$\left(\frac{\beta C_{lp}}{k}\right)_{C_L=0}$$

Datos experimentales

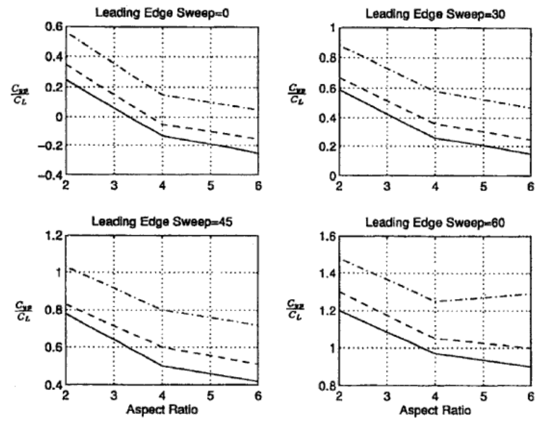


Fig. 4.24 The parameter $(C_{yp}/C_L)_{C_L=0,M=0}$ at supersonic speeds.[7]

Lateral-Directional C_{Y_p}

Método I

Ref: Pamadi

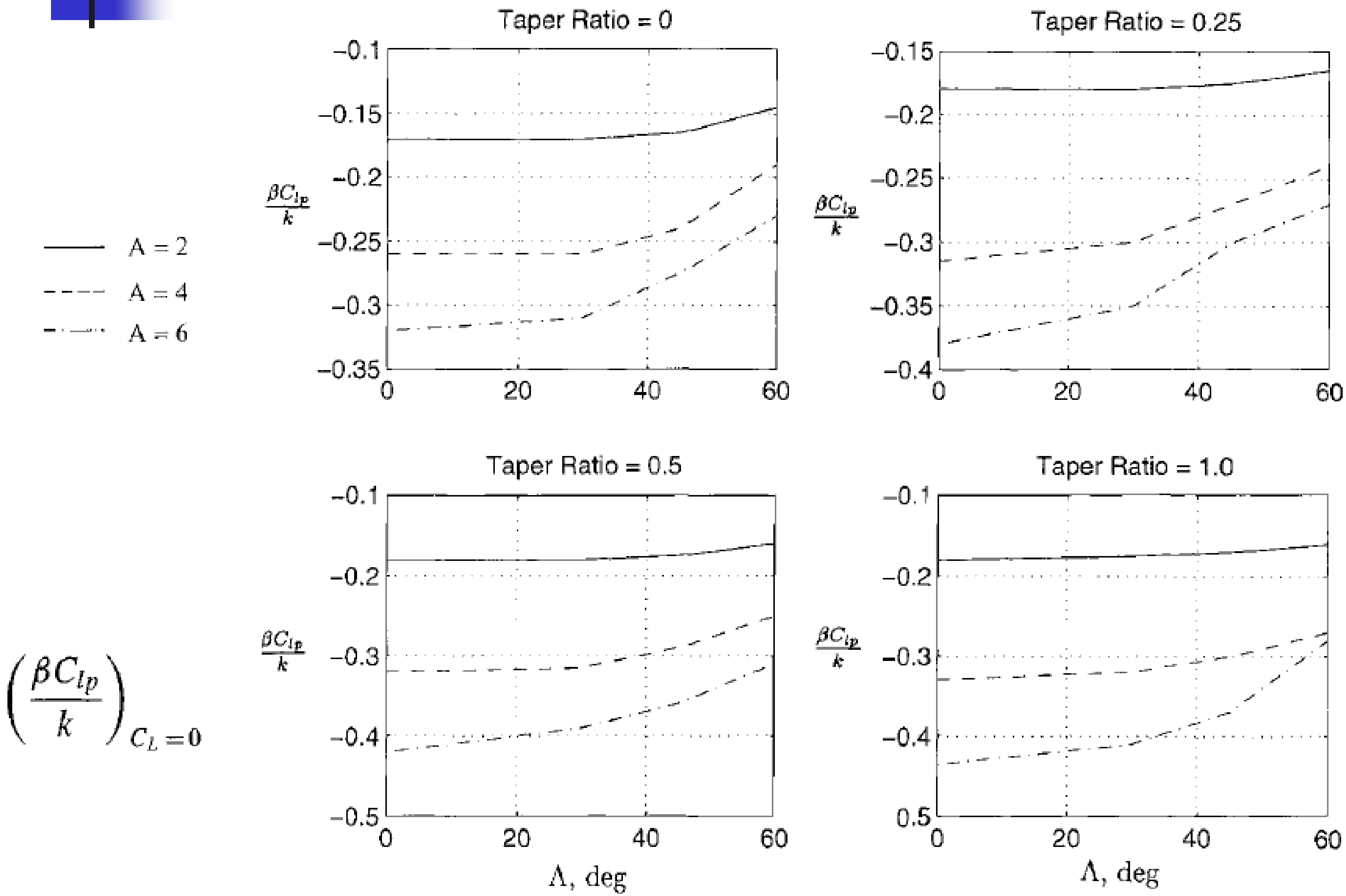


Fig. 4.25 **The parameter $(\beta C_{l_p}/k)_{C_L=0}$ at subsonic speeds.**[7]

Lateral-Directional C_{Y_p}

Método II

Ref: DarCorp

Método II → Considera la contribución del ala y del vertical

The airplane sideforce-coefficient-due-to-roll-rate derivative is primarily influenced by the vertical tail

$$C_{y_p} = 2C_{y_{\beta_v}} \frac{\left(Z_{ac_v} - Z_{cg}\right)\cos\alpha - \left(X_{ac_v} - X_{cg}\right)\sin\alpha - \left(Z_{ac_v} - Z_{cg}\right)}{b_w} + 3\sin\Gamma_w \left[1 - 4\frac{\left(Z_{cg} - Z_{c_r/4_w}\right)}{b_w}\sin\Gamma_w\right] C_{l_p @ \Gamma_w = 0,\ C_L = 0}$$

where:

$C_{n_{\beta_v}}$	is the vertical tail contribution to the sideforce-coefficient-due-to-sideslip derivative.
Z_{ac_v}	is the Z-coordinate of the vertical tail aerodynamic center.
Z_{cg}	is the Z-coordinate of the airplane center of gravity.
$Z_{c_r/4_w}$	is the Z-coordinate of the wing root chord.
X_{ac_v}	is the X-coordinate of the vertical tail aerodynamic center.
X_{cg}	is the X-coordinate of the airplane center of gravity.
α	is the airplane angle of attack.
b_w	is the wing span.
Γ_w	is the wing dihedral angle.
$C_{l_p @ \Gamma_w = 0,\ C_L = 0}$	is the roll damping derivative at zero lift and withour dihedral.

Lateral-Directional C_{Y_p}

Método II

Ref: DarCorp

The roll damping derivative of the wing without dihedral and at zero lift is found from:

$$C_{l_p}\,{}_{\substack{@\Gamma_w=0\\ C_L=0}} = \frac{k}{\beta}\left(\frac{\beta C_{l_p}}{k}\right)_{@C_L=0}$$

The roll damping parameter at zero lift is found from Figure 10.35 in Airplane Design Part VI and is a function of the wing aspect ratio, the Prandtl-Glauert transformation factor, the sectional lift curve slope obtained through the Prandtl-Glauert transformation factor, the wing quarter chord sweep angle, and the taper ratio:

$$\left(\frac{\beta C_{l_p}}{k}\right)_{@C_L=0} = f\left(AR_w, \beta, k, \Lambda_{c/4_w}, \lambda_w\right)$$

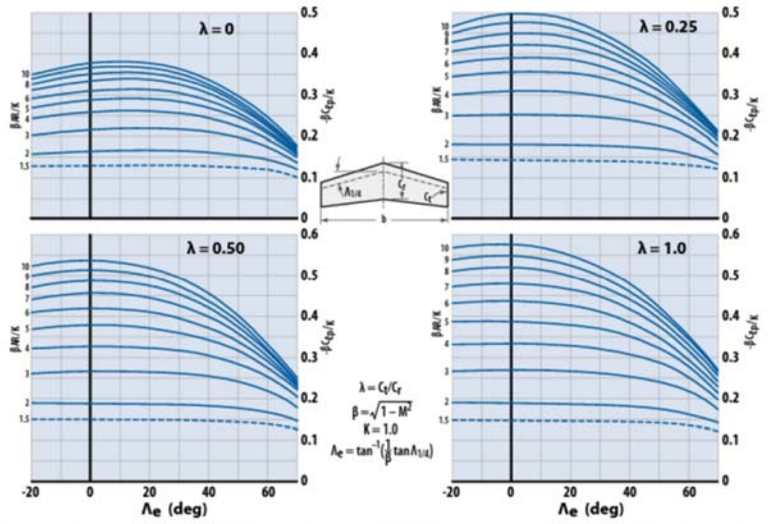


Figure 21.11 C_{l_p} for straight wings (data from [6]).

Figs A33

AR_w →wing aspect ratio.
β → Prandtl-Glauert transformation factor.
k → ratio of the incompressible sectional lift curve slope of the wing to 2π
$\Lambda_{c/4_w}$ → wing quarter chord sweep angle.
λ_w → wing taper ratio.

The Prandtl-Glauert transformation factor is defined as $\beta = \sqrt{1 - M_1^{\,2}}$

ratio of incompressible sectional lift curve slope with 2π $\quad k = \dfrac{f_{gap_{wo}}\, c_{l_{\alpha_w}}\,@M=0}{2\pi}$ → Aproximación $k = 1$

Fig A33

Método II

Ref: DarCorp

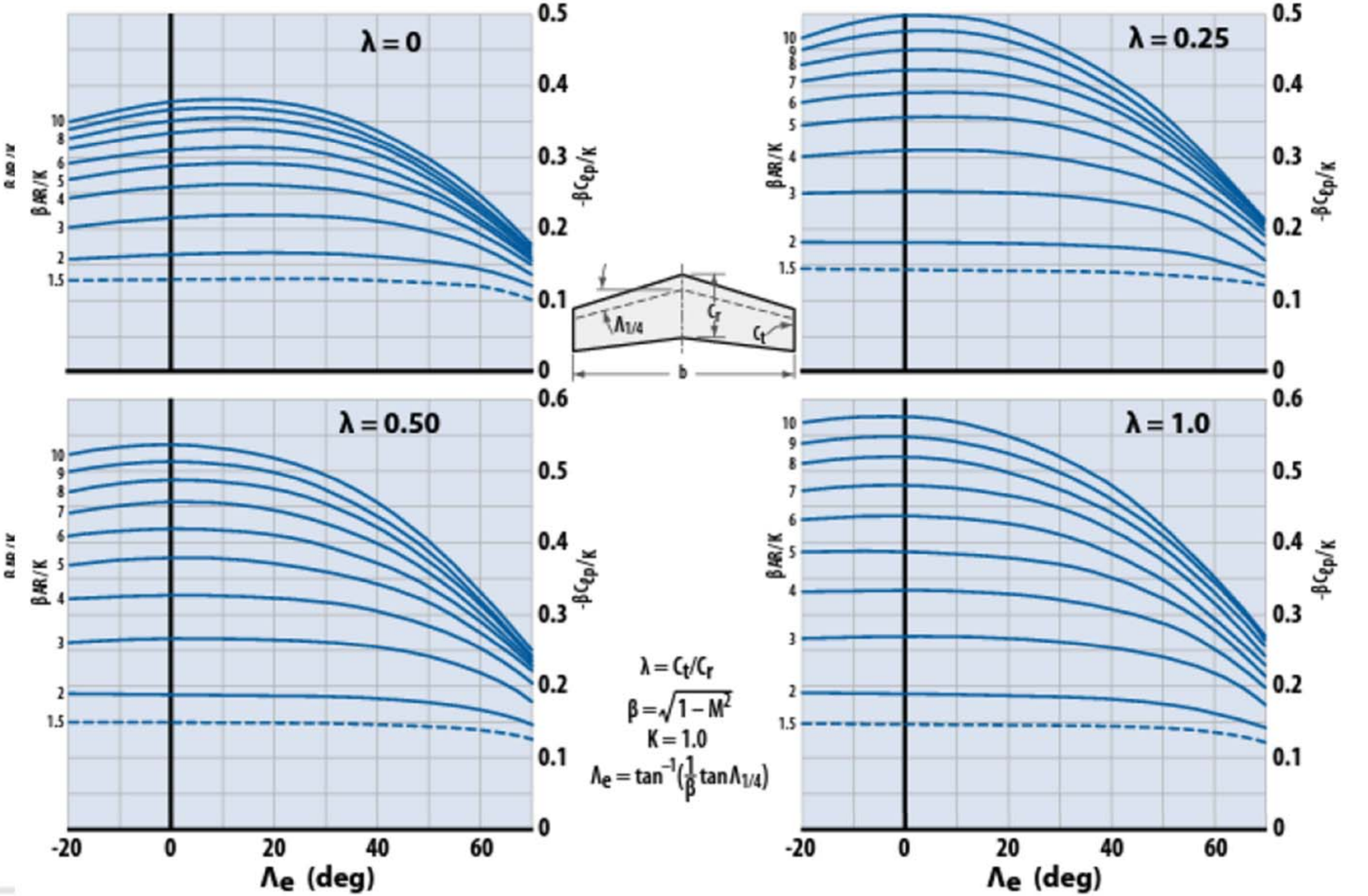


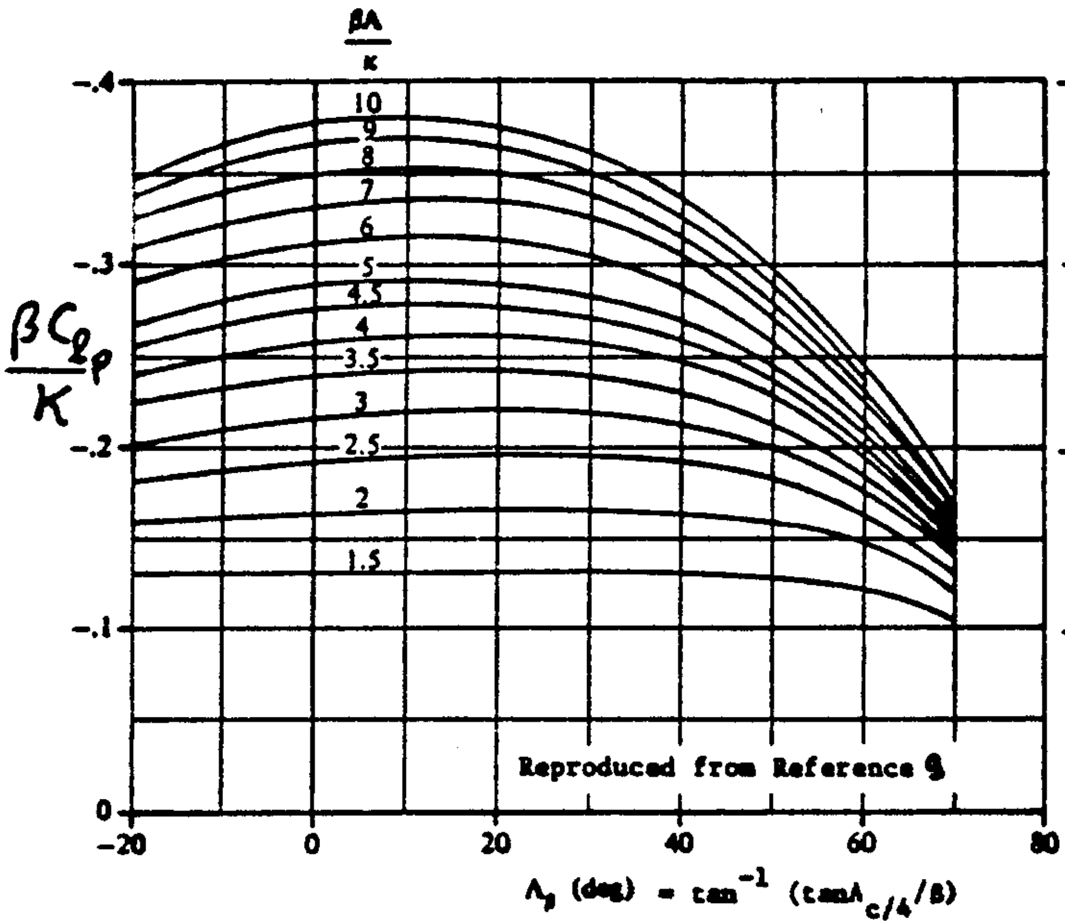
Figure 21.11 C_{ℓ_p} for straight wings (data from [6]).

Fig A33

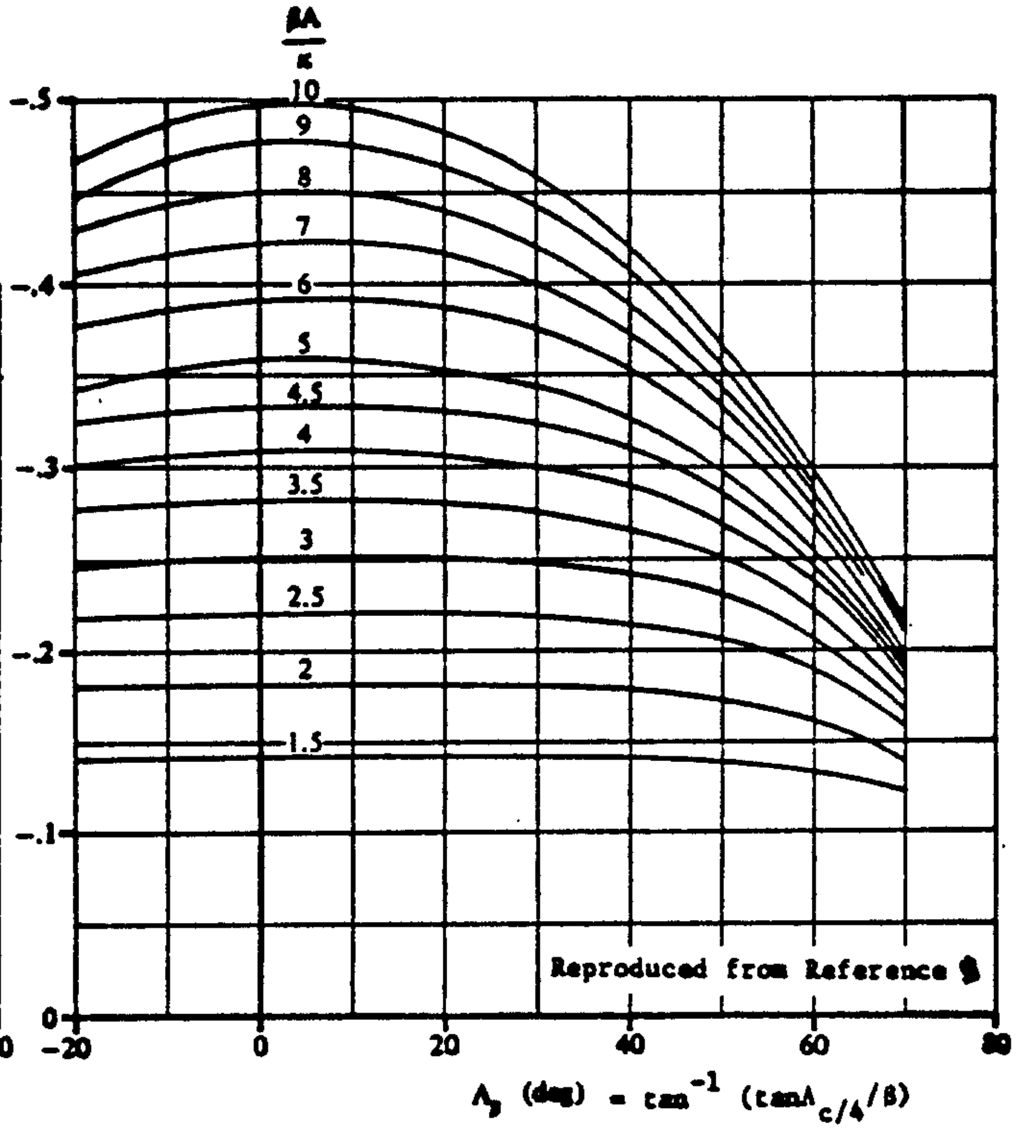
β : EQN. (10.53)

K : EQN. (10.54)

(a) $\lambda = 0$



(b) $\lambda = 0.25$

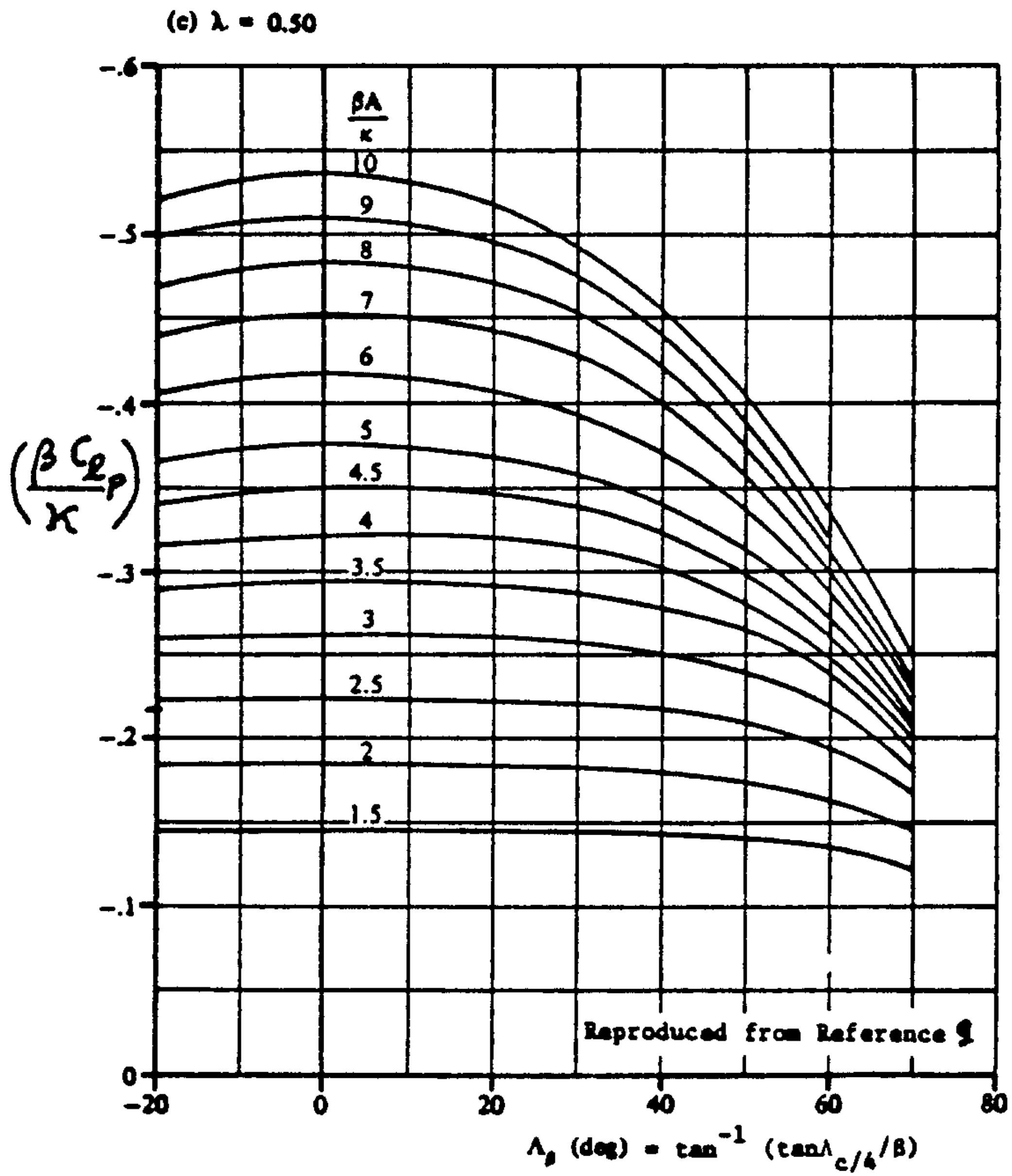


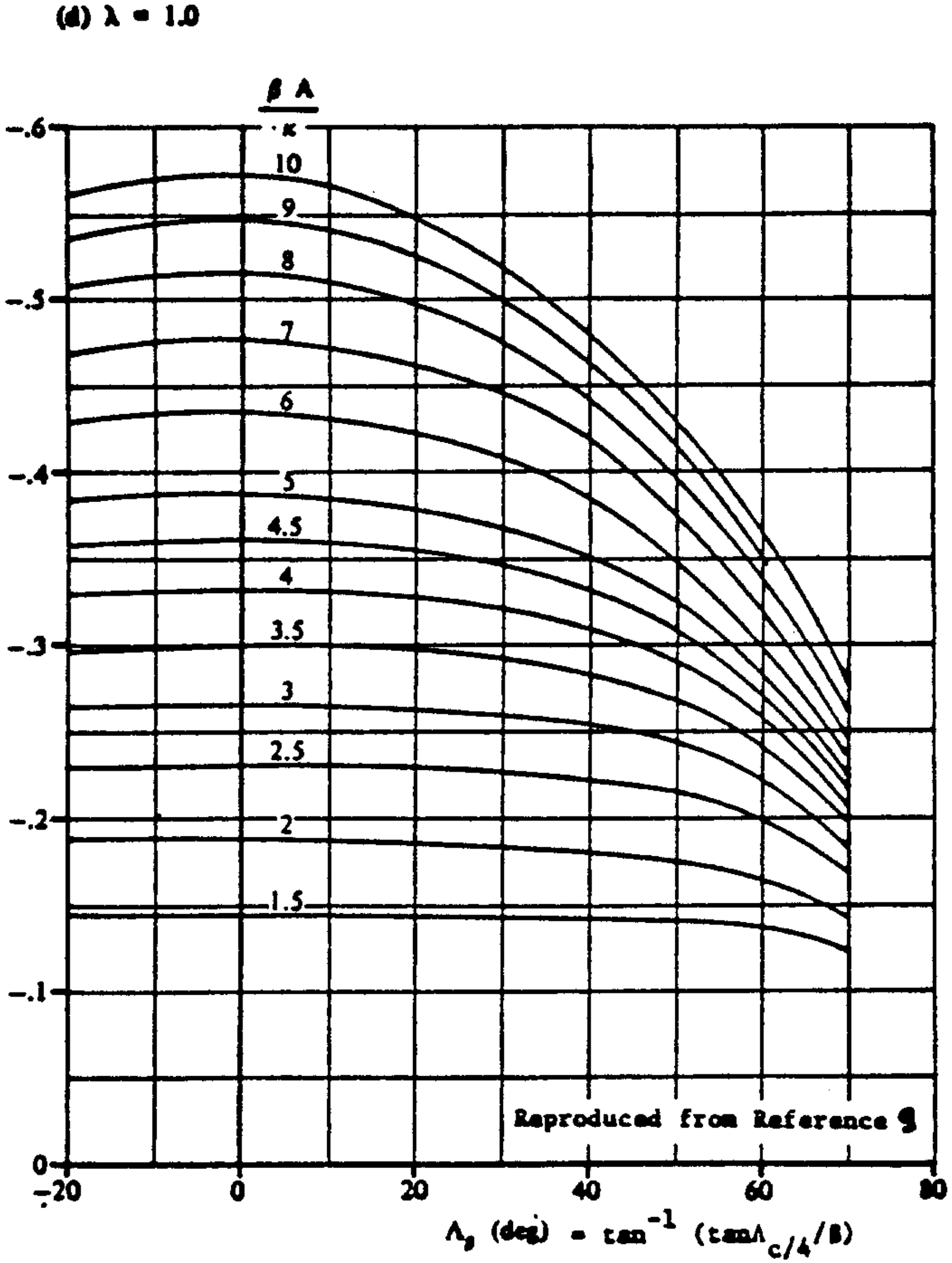
(d) $\lambda = 1.0$

Fig A33

Método II

Ref: DarCorp





Lateral-Directional C_{L_p}

Ref: Pamadi

Contribución total

ala

horizontal

V-tail

$$C_{l_p} = (C_{l_p})_W + (C_{l_p})_V + \left(C_{l_p}\right)_h + \left(C_{l_p}\right)_C + \left(C_{l_p}\right)_{vee}$$

vertical

canard

Contribución canard,horizontal y V-tail muy pequeñas → despreciable en 1ª aproximación

wing contribution

$$(C_{l_p})_W = \frac{-4}{Sb^2}\int_0^{\frac{b}{2}} [a_o(y) + C_{D,l}]c(y)y^2\,\mathrm{d}y$$

Strip theory

Aproximación

For an untwisted rectangular wing with a constant chord and an identical airfoil section along the span,

$a_0\ C_{l_\alpha}\ 2D$

$$(C_{l_p})_W = -\tfrac{1}{6}(a_o + C_D)$$

Strip theory ignores the induced drag effects and the mutual interference between adjacent wing sections → aproximación no válida para AR pequeños

Contribución Ala - I

C_{L_p}

Ref: Pamadi

Aproximación para AR normales

$$(C_{l_p})_W = \left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}}\right)\Big/\text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

$$\beta = \sqrt{1-M^2},$$

$$k = \frac{a_o}{2\pi}$$

$$\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}} = (1 - 2z' \sin\Gamma + 3z'^2 \sin^2\Gamma)/\text{rad}$$

$$z' = \frac{2z_w}{b}$$

z_w is the vertical distance between the center of gravity and the wing root chord, positive for center of gravity above the root chord.

$$\left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}$$

Datos experimentales

$\frac{\beta C_{l_p}}{k}$

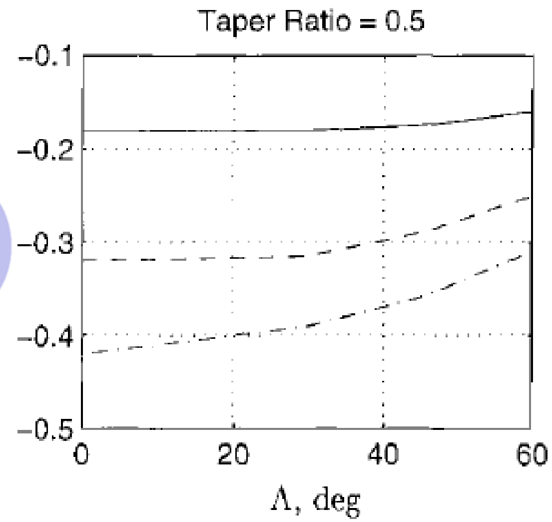


Fig A22

Fig A22

Ref: Pamadi

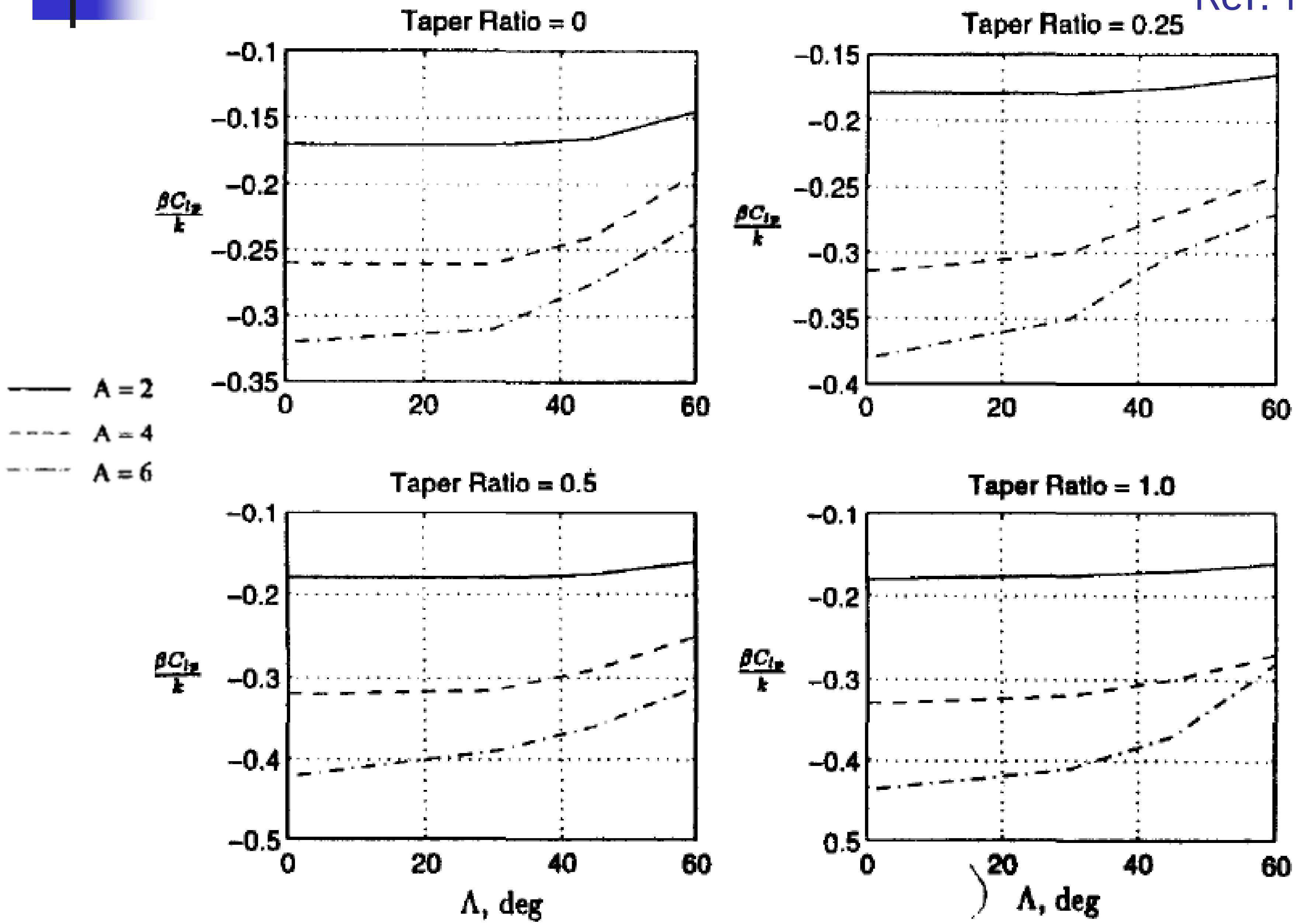


Fig. 4.25 The parameter $(\beta C_{l_p}/k)_{C_L=0}$ at subsonic speeds.[7]

Contribución Ala - II C_{L_p}

Ref: Pamadi & Roskam

Aproximación para AR normales

$$(C_{l_p})_W = \left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}}\right)\Big/\text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

$$\left[\left(\Delta C_{l_p}\right)_{drag}\right]_w = \frac{\left(C_{l_p}\right)_{C_{D_L}}}{C_{L_w}^2}\left(C_{L_w} + \Delta C_{L_{\delta f}}\right)^2 - 0.125\left(C_{D_{0w}} + C_{D_{0\,flap}}\right)$$

$\dfrac{\left(C_{l_p}\right)_{C_{D_L}}}{C_{L_{l.s.}}^2}$ ➡ is the wing drag-due-to-lift roll damping parameter. ➡ **Fig. C13**

C_{L_w} - is the wing lift coefficient without any flap effects.

$\Delta C_{L_{\delta f}}$ - is the lift coefficient due to flap deflection.

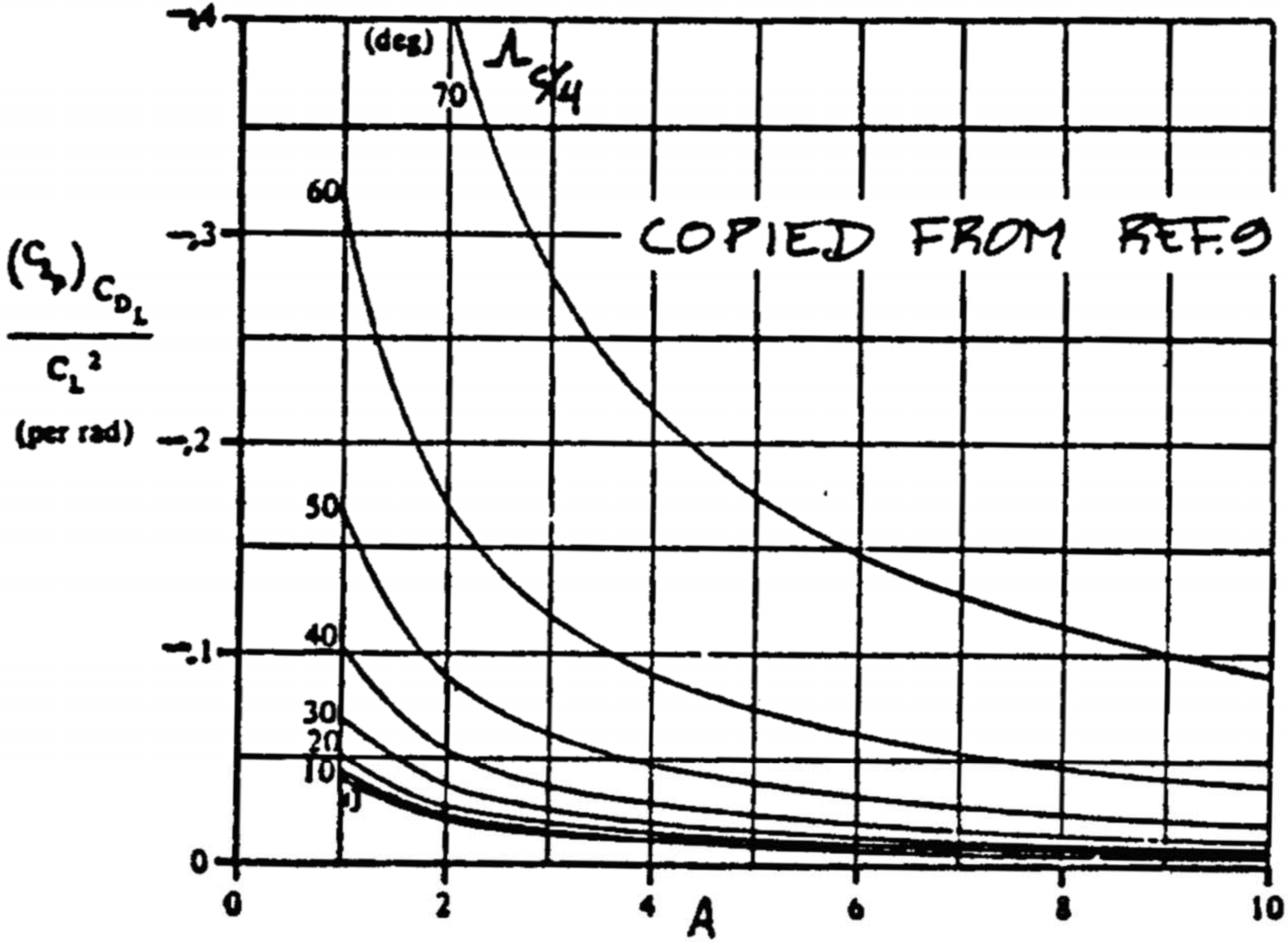
$C_{D_{0_w}}$ - is the wing zero-lift drag coefficient.

$C_{D_{0_{flap}}}$ - is the flap profile and interference drag coefficient.

Fig. C13

Ref: Pamadi & Roskam

drag-due-to-lift roll damping parameter – En función de $\Lambda_{c/4}$



Contribución Vertical C_{L_p}

Ref: Pamadi

Tail contribution

$$(C_{lp})_V = \left|2\left(\frac{z}{b}\right)\left(\frac{z - z_v}{b}\right)\right| C_{y\beta,V}$$

$$z = z_v \cos\alpha - l_v \sin\alpha$$

α ángulo de ataque de trimado
asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline

$$l_v = X_{ac_v} - X_{cg} \qquad z_v = Z_{ac_v} - Z_{cg}$$

$$C_{y\beta,V} = -ka_v\left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v\frac{S_v}{S}$$

Contribución (Canard/Horizontal/V-tail) C_{L_p}

Método I

Contribución horizontal & V-tail y canard (misma ecuación, utilizar proyección horizontal V-tail)

$$C_{l_{p\,l.s.}} = \frac{1}{2}\left(C_{l_p}\right)_{l.s.}\frac{S_{l.s.}}{S_w}\left(\frac{b_{l.s.}}{b_w}\right)^2$$

$S_{l.s.}$ superficie lifting surface → Canard, Horizontal, V-tail

S_w superficie alar

$b_{l.s}$ envergadura lifting surface

b_w envergadura ala

El subíndice l.s. se substituye por el de la superficie que se esté evaluando

El valor de $\left(C_{l_p}\right)_{l.s.}$ es el que se obtiene para cada una de las distintas superficies aerodinámicas utilizando las mismas ecuaciones para el cálculo de la contribución del ala

$$\left(C_{l_p}\right)_{l.s.} \Rightarrow (C_{l_p})_W = \left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}}\right) \Big/ \text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

Ref: Roskam

emplear ecuaciones para contribución ala

Contribución Horizontal

C_{L_p}

Método I

Contribución horizontal $l.s. \rightarrow h$

$$C_{l_{p\,l.s.}} = \frac{1}{2}\left(C_{l_p}\right)_{l.s.} \frac{S_{l.s.}}{S_w}\left(\frac{b_{l.s.}}{b_w}\right)^2$$

$$\left(C_{l_p}\right)_{l.s.} \Rightarrow (C_{lp})_W = \left(\frac{\beta C_{lp}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{lp})_\Gamma}{(C_{lp})_{\Gamma=0}}\right)\Big/\text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

$$\left[\left(\Delta C_{l_p}\right)_{drag}\right]_h = \frac{\left(C_{l_p}\right)_{C_{D_L}}}{C_{L_h}^2} C_{L_h}^2 - 0.125 C_{D_{0h}}$$

$\left(\Delta C_{l_p}\right)_{drag_h}$ - is the horizontal tail lift coefficient.

C_{L_h} - is the horizontal tail lift coefficient.

$C_{D_{0_h}}$ - is the horizontal tail zero-lift drag coefficient.

Ref: Roskam

C_{L_p}

Método I

Contribución V-tail $l.s. \rightarrow vee$

$$C_{l_{p_{l.s.}}} = \frac{1}{2}\left(C_{l_p}\right)_{l.s.} \frac{S_{l.s.}}{S_w}\left(\frac{b_{l.s.}}{b_w}\right)^2$$

$$\left(C_{l_p}\right)_{l.s.} \Rightarrow (C_{l_p})_W = \left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}}\right) \Big/ \text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

$$\left[\left(\Delta C_{l_p}\right)_{drag}\right]_{vee} = \frac{\left(C_{l_p}\right)_{C_{D_L}}}{C^2_{L_{vee}}} C^2_{L_{vee}} - 0.125 C_{D_{0_{vee}}}$$

$\left(\Delta C_{l_p}\right)_{drag_{vee}}$ - is the V-Tail drag-due-to-lift roll damping parameter.

$C_{D_{0_{vee}}}$ - is the V-Tail lift coefficient.

$C_{D_{0_{vee}}}$ - is the V-Tail zero-lift drag coefficient.

Ref: Roskam

Contribución Canard

C_{L_p}

Contribución Canard $l.s. \rightarrow c$

Método I

$$C_{l_{p_{l.s.}}} = \frac{1}{2}\left(C_{l_p}\right)_{l.s.} \frac{S_{l.s.}}{S_w}\left(\frac{b_{l.s.}}{b_w}\right)^2$$

$$\left(C_{l_p}\right)_{l.s.} \Rightarrow (C_{l_p})_W = \left(\frac{\beta C_{l_p}}{k}\right)_{C_L=0}\left(\frac{k}{\beta}\right)\left(\frac{(C_{l_p})_\Gamma}{(C_{l_p})_{\Gamma=0}}\right)\Big/\text{rad} + \left[\left(\Delta C_{l_p}\right)_{drag}\right]_{l.s.}$$

$$\left[\left(\Delta C_{l_p}\right)_{drag}\right]_c = \frac{\left(C_{l_p}\right)_{C_{D_L}}}{C_{L_c}^2} C_{L_c}^2 - 0.125 C_{D_{0_c}}$$

$\left(\Delta C_{l_p}\right)_{drag_c}$ - is the canard drag-due-to-lift roll damping parameter.

C_{L_c} - is the canard lift coefficient.

$C_{D_{0_c}}$ - is the canard zero-lift drag coefficient.

Ref: Roskam

Lateral-Directional C_{N_p}

Ref: Pamadi

Contribución total

ala

$$C_{np} = (C_{np})_W + (C_{np})_V$$

vertical

wing contribution

$$(C_{np})_W = \frac{-4}{Sb^2}\int_0^{b/2}[a_o(y)\alpha - C_{D\alpha,l}]c(y)y^2\,dy$$

Strip theory

Aproximación

For an untwisted rectangular wing with a constant chord and an identical airfoil section along the span,

$$(C_{np})_W = -\tfrac{1}{6}(C_L - C_{D\alpha})/\text{rad}$$

Strip theory ignores the induced drag effects and the mutual interference between adjacent wing sections → aproximación no válida para AR pequeños

Contribución Ala - I

C_{N_p}

Ref: Pamadi

wing contribution

Aproximación más precisa (velocidades subsónicas)

$$(C_{np})_W = C_{lp} \tan\alpha (K-1) + K \left(\frac{C_{np}}{C_L}\right)_{C_L=0,M} \quad C_L/\text{rad}$$

$$K = \frac{1-a_{w1}}{1-a_{w2}} \Rightarrow \quad a_{w2} = e a_{w1} \qquad a_{w1} = \frac{(C_{L\alpha})_e}{\pi A e}$$

$(C_{L\alpha})_e \Rightarrow$ Pendiente de sustentación del ala expuesta

En 1ª aproximación $\left(C_{L_\alpha}\right)_e \approx C_{L_\alpha}$ del ala

Coeficiente de Oswald (dept. aerodinámica) $\Rightarrow$

$$e = \frac{1.1 C_{L\alpha}}{R C_{L\alpha} + (1-R)\pi A} \Rightarrow R = a_1\lambda_1^3 + a_2\lambda_1^2 + a_3\lambda_1 + a_4$$

$$a_1 = 0.0004, \quad a_2 = -0.0080, \quad a_3 = 0.0501, \quad a_4 = 0.8642, \qquad \lambda_1 = A\lambda / \cos\Lambda_{LE}$$

A is the aspect ratio, λ is the taper ratio, and Λ_{LE} is the leading-edge sweep of the wing.

Contribución Ala - II

C_{N_p}

wing contribution

Ref: Pamadi

Aproximación más precisa (velocidades subsónicas)

$$(C_{np})_W = C_{lp}\tan\alpha(K-1) + K\left(\frac{C_{np}}{C_L}\right)_{C_L=0,M} C_L/\text{rad}$$

$$\left(\frac{C_{np}}{C_L}\right)_{C_L=0,M} = \left(\frac{A+4\cos\Lambda_{c/4}}{AB+4\cos\Lambda_{c/4}}\right)\left[\frac{AB+0.5(AB+\cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}{A+0.5(A+\cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}\right]\times\left(\frac{C_{np}}{C_L}\right)_{C_L=M=0}\Big/\text{rad}$$

$$B = \sqrt{1-M^2\cos^2\Lambda_{c/4}}$$

A is the exposed aspect ratio (A_e)

$$\left(\frac{C_{np}}{C_L}\right)_{C_L=M=0} = -\left[\frac{A+6(A+\cos\Lambda_{c/4})\left(\frac{\xi\tan\Lambda_{c/4}}{A}+\frac{\tan^2\Lambda_{c/4}}{12}\right)}{6(A+4\cos\Lambda_{c/4})}\right]$$

Contribución Ala - III

C_{N_p}

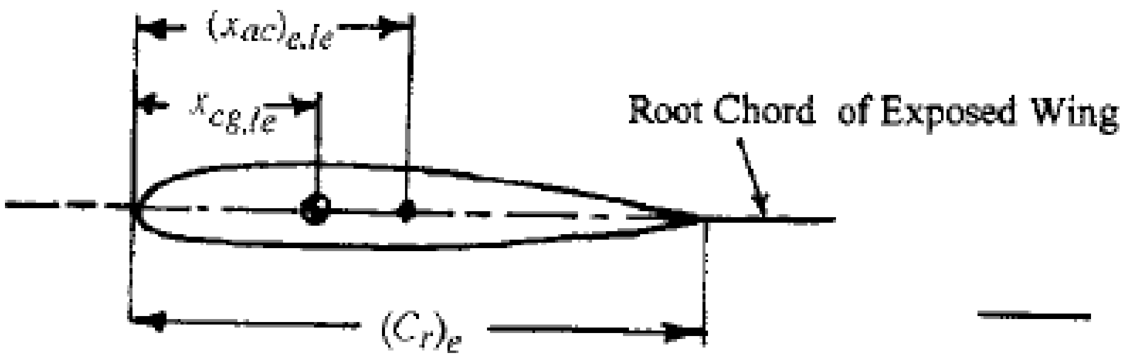
Ref: Pamadi

wing contribution

$$\xi = \frac{\bar{x}}{\bar{c}_e} \quad \Rightarrow \quad \bar{x} = (x_{ac})_e - x_{cg,le} \quad \Rightarrow$$ ≈SM (márgen estático)

$(x_{ac})_e$ is the distance of the exposed wing aerodynamic center from the leading edge of the root chord, $x_{cg,le}$ is the distance of the center of gravity from the leading edge of the exposed wing root chord. Both $(x_{ac})_e$ and $x_{cg,le}$ are measured parallel to the exposed wing root chord.

The parameter $\bar{x}$ will be positive if the aerodynamic center of the exposed wing $(x_{ac})_e$ is aft of the center of gravity



Contribución Vertical C_{N_p}

Ref: Pamadi

Vertical contribution

$$(C_{np})_V = -\left(\frac{2}{b}\right)(l_v \cos\alpha + z_v \sin\alpha)\left(\frac{z - z_v}{b}\right) C_{y\beta,V} \,/\mathrm{rad}$$

$$z = z_v \cos\alpha - l_v \sin\alpha$$

α ángulo de ataque de trimado
asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline

$$l_v = X_{ac_v} - X_{cg} \qquad z_v = Z_{ac_v} - Z_{cg}$$

$$C_{y\beta,V} = -k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v \frac{S_v}{S}$$

Derivadas $C_{Y_r}, C_{L_r}, C_{N_r}$

Estimación Derivadas

- Contribución C_{Y_r}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje
- Contribución C_{L_r}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje
  - Horizontal/Canard/V-tail
- Contribución C_{N_r}
  - Ala: flecha, diedro
  - Vertical
  - Fuselaje

Ref: Pamadi/Roskam

Total contribution

ala

$$C_{y_r} = \left(C_{y_r}\right)_w + \left(C_{y_r}\right)_v$$

vertical

wing contribution: The wing contribution is normally negligible, as indicated by TN-1669

$$\left(C_{y_r}\right)_w = 0.143\,C_L - 0.05$$

Aproximación alas rectangulares y flecha zero

tail contribution

$$(C_{y_r})_V = -\frac{2}{b}(l_v \cos\alpha + z_v \sin\alpha) C_{y\beta,V}/\text{rad}$$

$$C_{y\beta,V} = -k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v\left(\frac{S_v}{S}\right)$$

$$\left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v = 0.724 + \frac{3.06 S_v/S}{1+\cos\Lambda_{c/4}} + \frac{0.4 z_w}{d_{f,\max}} + 0.009A$$

$$a_v = a_w = \frac{2\pi A}{2 + \sqrt{\frac{A^2\beta^2}{k^2}\left(1 + \frac{\tan^2\Lambda_{c/2}}{\beta^2}\right) + 4}}$$

Contribución Ala

C_{L_r}

Ref: Pamadi/Roskam

Total contribution

ala

$$C_{l_r} = \left(C_{l_r}\right)_w + \left(C_{l_r}\right)_v$$

vertical

Wing contribution

$$(C_{lr})_W = \frac{8}{Sb^2} \int_0^{\frac{b}{2}} a_o(y)\alpha c(y) y^2 \, \mathrm{d}y$$

For an untwisted rectangular wing of constant chord and constant airfoil section all along the span,

Aproximación

$$(C_{lr})_W = \frac{C_L}{3}$$

Strip theory ignores the induced drag effects and the mutual interference between adjacent wing sections → aproximación no válida para AR pequeños

Contribución Ala

C_{L_r}

Wing contribution

Ref: DarCorp

ala

Wing twist

$$C_{l_{r_w}} = \left(C_{L_{w\,clean}}\right)\left(\frac{C_{l_r}}{C_L}\right)_{\substack{@C_L=0\\M_1}} + \frac{\Delta C_{l_r}}{\Gamma_w}\Gamma_w + \frac{\Delta C_{l_r}}{\varepsilon_{t_w}}\varepsilon_{t_w} + \frac{\Delta C_{l_r}}{\alpha_{\delta_f}\delta_f}\alpha_{\delta_f}\delta_f$$

diedro

flap

where:

$C_{L_{w\,clean}}$	is the wing lift coefficient without any flap effects.
$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{@C_L=0\\M_1}}$	is the slope of the rolling moment coefficient due to roll rate at zero lift.
$\frac{\Delta C_{l_r}}{\Gamma_w}$	is the dihedral contribution to the rolling-moment-coefficient-due-to-yaw-rate derivative.
Γ_w	is the wing dihedral angle.
$\frac{\Delta C_{l_r}}{\varepsilon_{t_w}}$	is the increment in the rolling-moment-coefficient-due-to-yaw- rate derivative due to wing twist.
ε_{t_w}	is the wing twist angle.
$\frac{\Delta C_{l_r}}{\alpha_{\delta_f}\delta_f}$	is the effect of symmetric flap deflection on the rolling-moment-coefficient-due-to-yaw-rate derivative.
α_{δ_f}	is the change in airplane angle of attack due to flap deflection.
δ_f	is the flap deflection angle.

Contribución Ala

$$C_{L_r}$$

Ref: Pamadi

The slope of the rolling-moment-due-to-roll-rate at zero-lift

$$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{@C_L=0\\ M_1}} \Rightarrow \left(\frac{C_{l_r}}{C_L}\right)_{C_L=0,M} = \frac{\text{Num}}{\text{Den}}\left(\frac{C_{l_r}}{C_L}\right)_{C_L=0,M=0}$$

$$\text{Num} = 1 + \frac{A(1-B^2)}{2B(AB+2\cos\Lambda_{c/4})} + \left(\frac{AB+2\cos\Lambda_{c/4}}{AB+4\cos\Lambda_{c/4}}\right)\left(\frac{\tan^2\Lambda_{c/4}}{8}\right)$$

$$\text{Den} = 1 + \left(\frac{A+2\cos\Lambda_{c/4}}{A+4\cos\Lambda_{c/4}}\right)\left(\frac{\tan^2\Lambda_{c/4}}{8}\right)$$

$$\left(\frac{\Delta C_{l_r}}{\Gamma}\right) = \left(\frac{1}{12}\right)\left(\frac{\pi A\sin\Lambda_{c/4}}{A+4\cos\Lambda_{c/4}}\right)\Big/\text{rad}^2$$

$A = A_e$, the exposed wing aspect ratio.

Fig A23

$$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0\\ M=0}} \Rightarrow$$

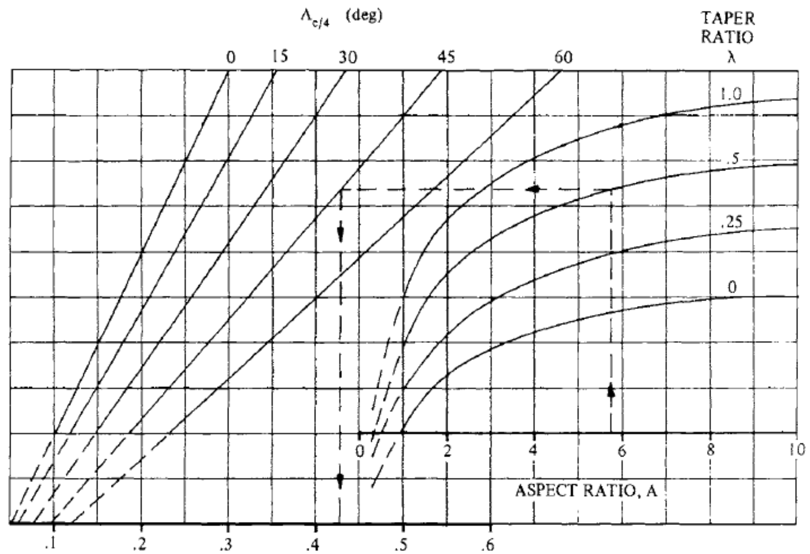


Fig A23

C_{L_r}

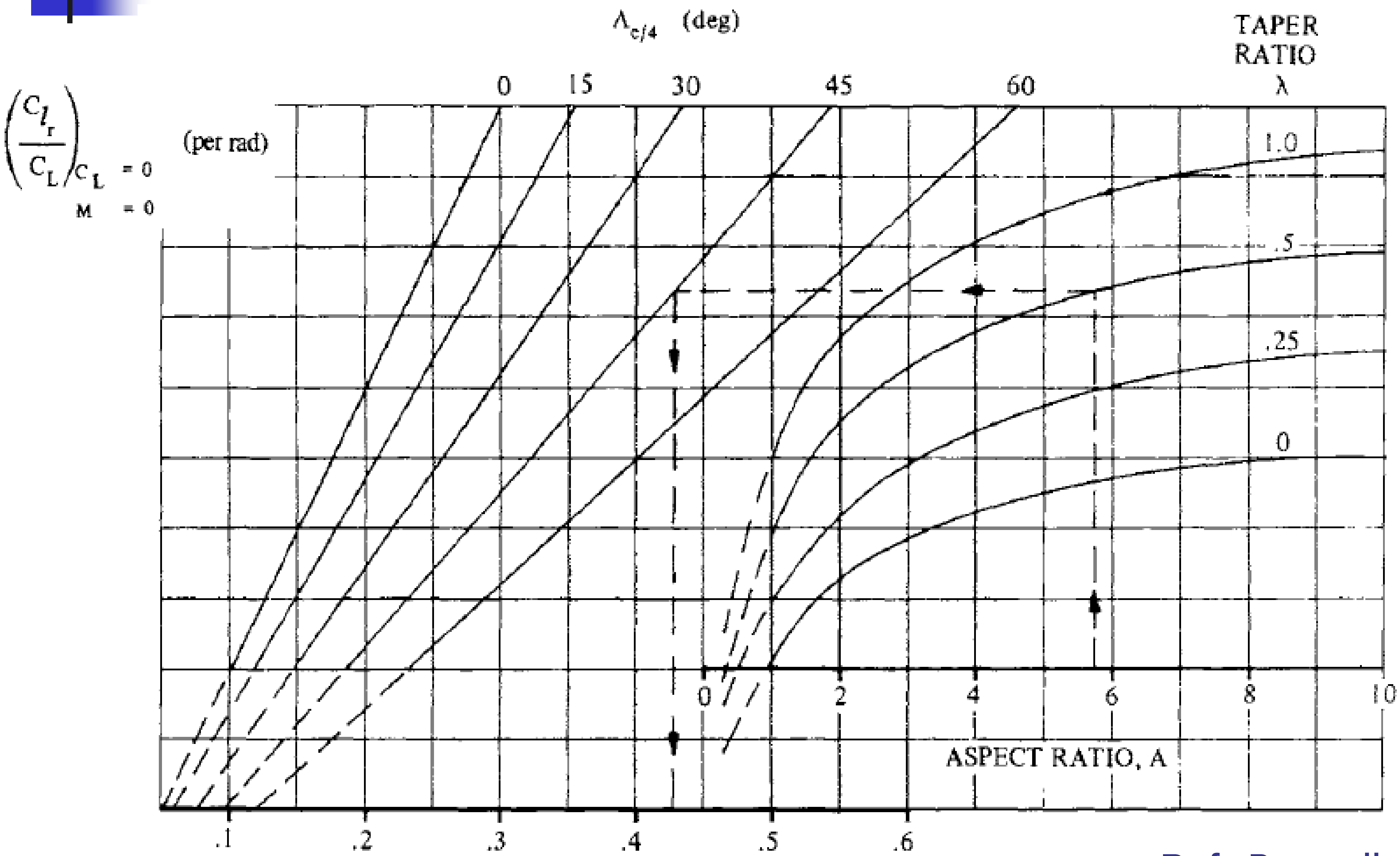


Fig. 4.28 The parameter $(C_{l_r}/C_L)_W$ for subsonic speeds.[7]

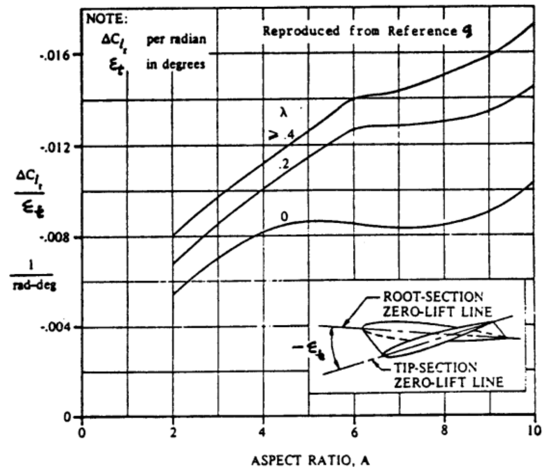
Ref: Pamadi

Contribución Ala

The increment in the rolling-moment-coefficient-due-to-yaw-rate due to wing twist is determined from Figure 10.42 in *Airplane Design Part VI* and is a function of wing aspect ratio and wing taper ratio

$$\frac{\Delta C_{l_r}}{\varepsilon_{t_W}} = f(AR_W, \lambda_W)$$

Fig A34



The effect of symmetric flap deflection on the rolling-moment-coefficient-due-to-yaw-rate derivative is obtained from Figure 10.43 in Airplane Design Partt VI and is a function of wing aspect ratio, wing taper ratio, inboard and outboard flap locations in terms of wing half span, and wing span:

$$\frac{\Delta C_{l_r}}{\alpha_{\delta_f}\delta_f} = f(AR_W, \lambda_W, \eta_{i_f}, \eta_{o_f}, b_W)$$

Fig A35

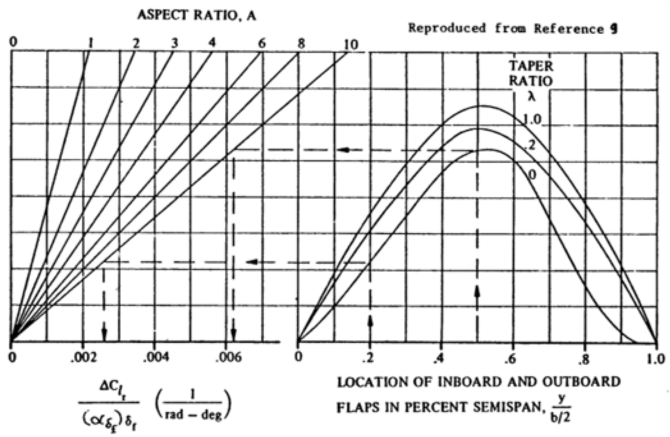


Fig A34

Ref: DarCorp C_{L_r}

$$\frac{\Delta C_{l_r}}{\varepsilon_{t_W}} = f(AR_W, \lambda_W)$$

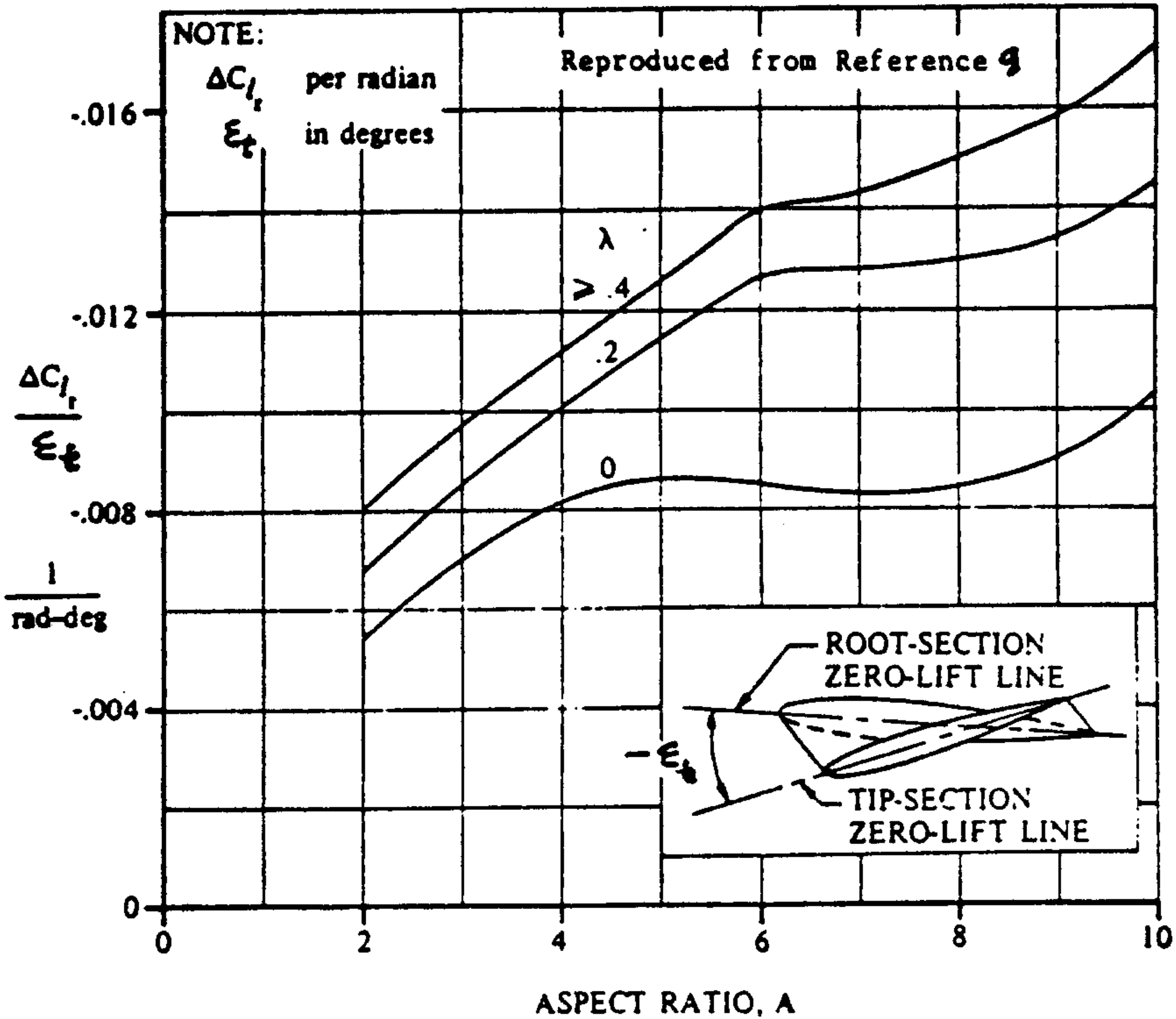


Fig A35

Ref: DarCorp

C_{L_r}

NOTE:

ΔC_{l_r} per radian

δ_f in degrees

NOTE: $$\frac{\Delta C_{l_r}}{(\alpha_{\delta_f})\,\delta_f} = \left[\frac{\Delta C_{l_r}}{(\alpha_{\delta_f})\,\delta_f}\right]_{\text{outboard}} - \left[\frac{\Delta C_{l_r}}{(\alpha_{\delta_f})\,\delta_f}\right]_{\text{inboard}}$$

Contribución Vertical C_{L_r}

Tail contribution

$$(C_{lr})_V = \frac{-2}{b^2}(l_v \cos\alpha + z_v \sin\alpha)(z_v \cos\alpha - l_v \sin\alpha) C_{y\beta,V}$$

α ángulo de ataque de trimado asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline

$$l_v = X_{ac_v} - X_{cg} \qquad z_v = Z_{ac_v} - Z_{cg}$$

$$C_{y\beta,V} = -k a_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right) \eta_v \frac{S_v}{S}$$

Contribución Ala

C_{N_r}

Ref: Pamadi/Roskam

Total contribution

ala

$$C_{n_r} = (C_{n_r})_w + (C_{n_r})_v$$

vertical

Wing contribution

$$(C_{nr})_W = \frac{-8}{Sb^2}\int_0^{\frac{b}{2}} C_{D,l}\,c(y)y^2\,\mathrm{d}y$$

all along the span,

Aproximación

$$(C_{nr})_W = -\frac{1}{3}C_{D,l}$$

Strip theory ignores the induced drag effects and the mutual interference between adjacent wing sections → aproximación no válida para AR pequeños

Contribución Ala - II

C_{N_r}

Wing contribution

Ref: Pamadi/Roskam

Fig A24

$$(C_{nr})_W = \left(\frac{C_{nr}}{C_L^2}\right) C_L^2 + \left(\frac{C_{nr}}{C_{DO}}\right) C_{DO} \Big/ \text{rad}$$

Fig A24

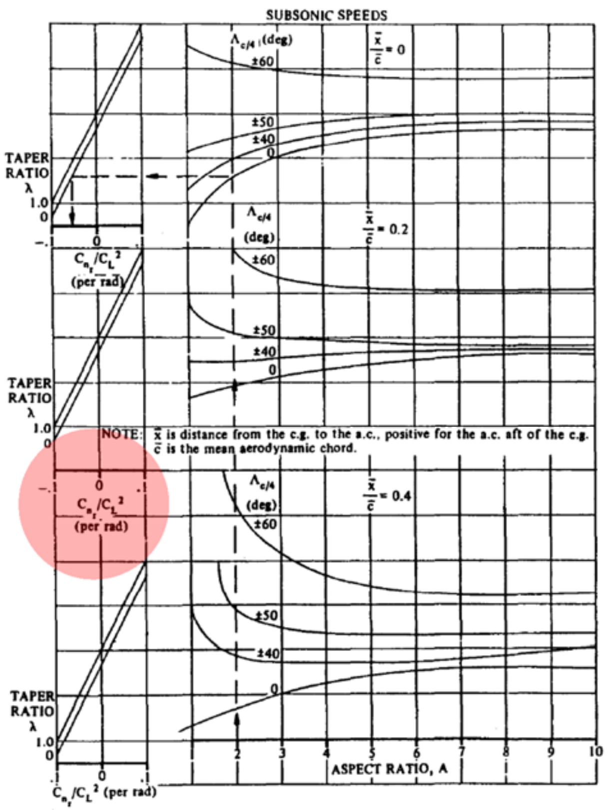


Fig. 4.29a The parameter $(C_{n_r}/C_L^2)_W$ for subsonic speeds.[7]

$\frac{\bar{x}}{\bar{c}}$: es función del SM

Fig A25

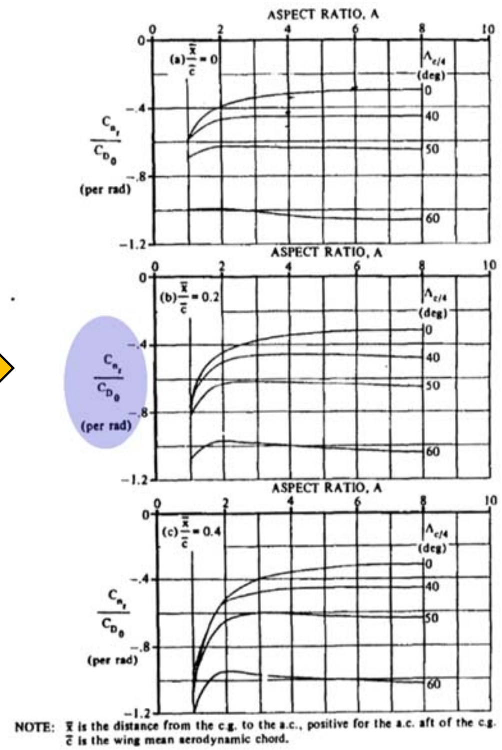


Fig. 4.29b The parameter $(C_{n_r}/C_{DO})_W$ for subsonic speeds.[7]

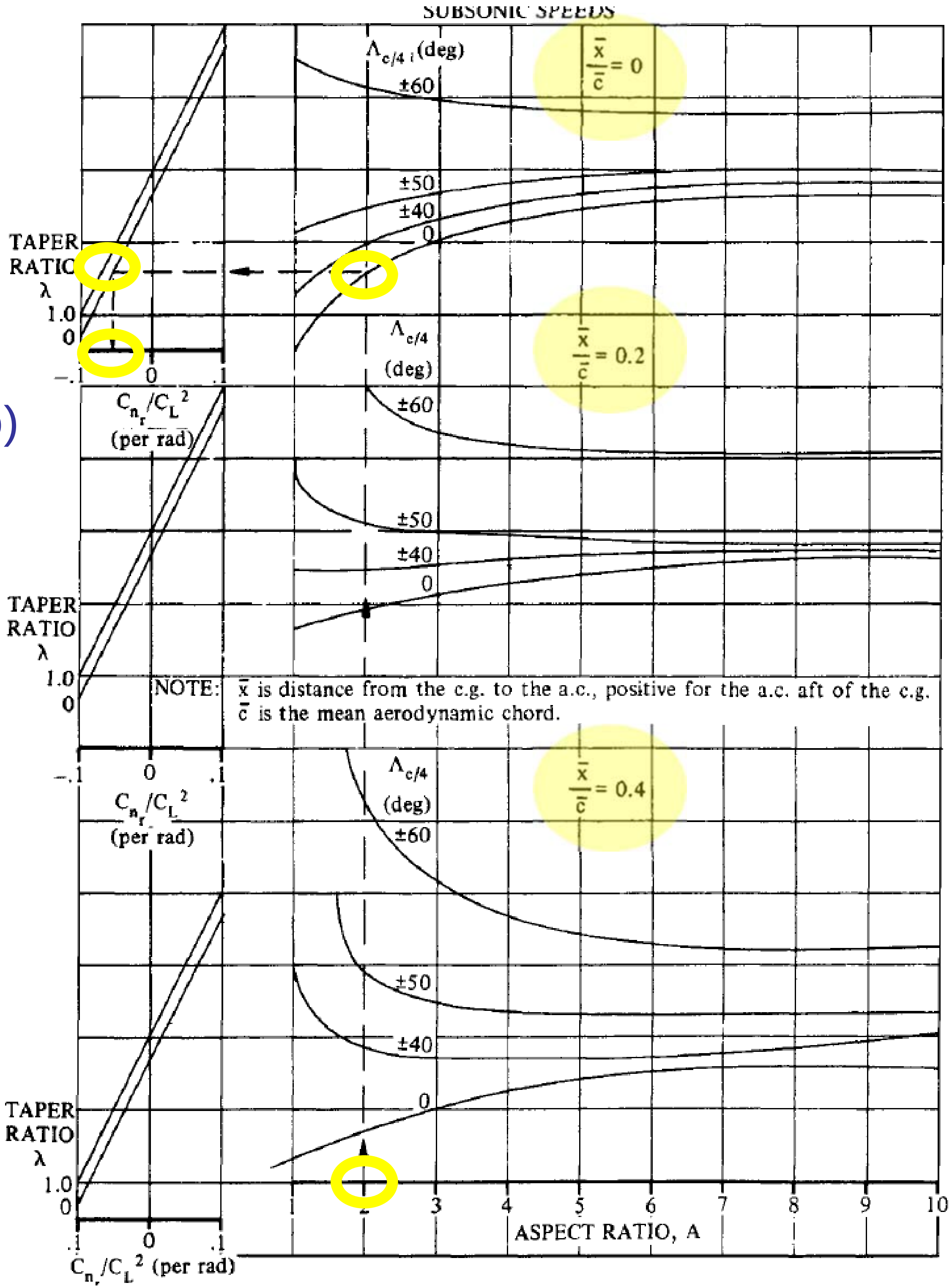
Fig A24

SM (Margen estático) = 0.0

$\frac{\bar{x}}{\bar{c}}$: función del SM (Margen estático)

SM (Margen estático) = 0.2

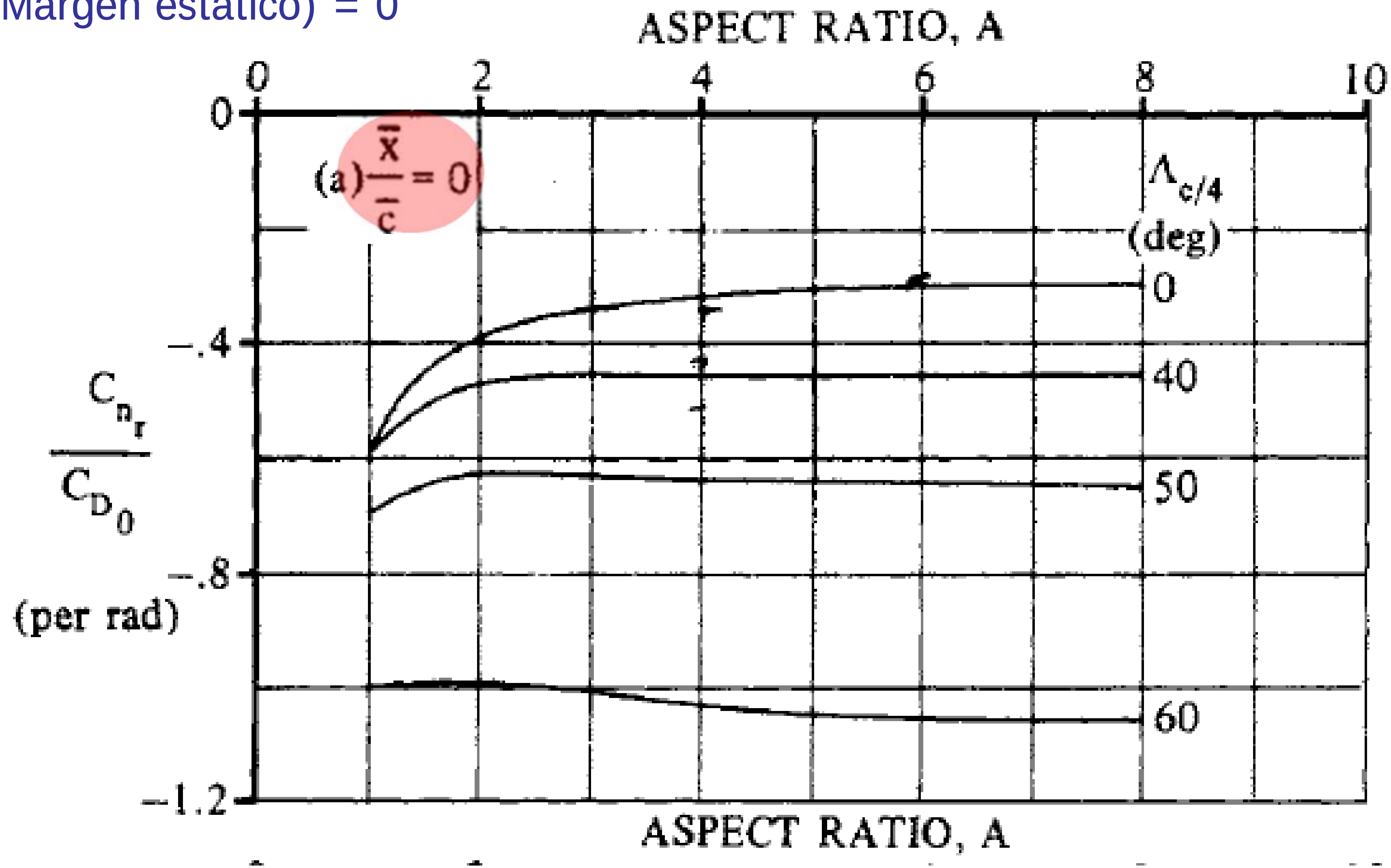
SM (Margen estático) = 0.4



Ref: Pamadi

Fig A25 - I

SM (Margen estático) = 0



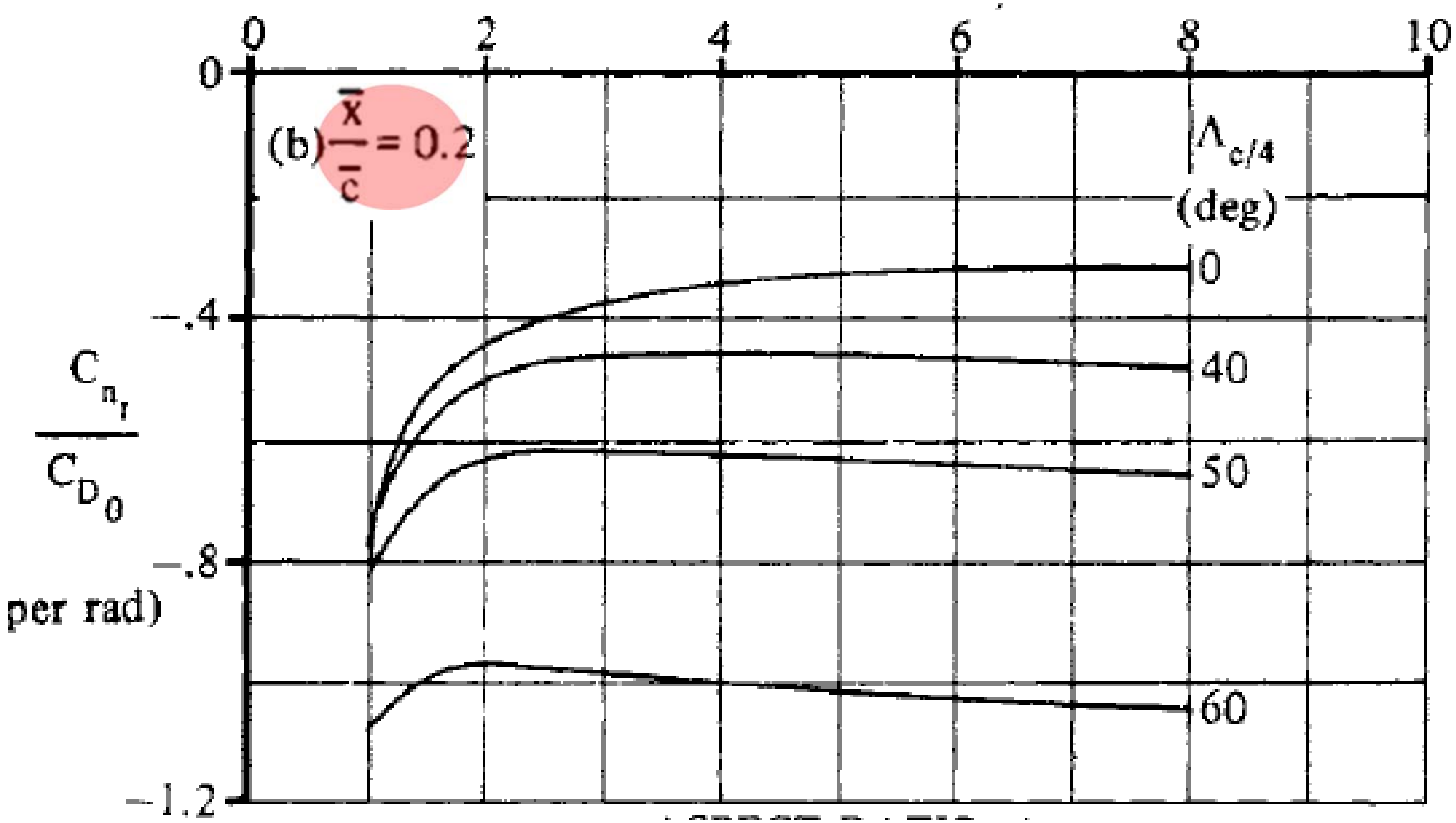
NOTE: $\bar{x}$ is the distance from the c.g. to the a.c., positive for the a.c. aft of the c.g.
$\bar{c}$ is the wing mean aerodynamic chord.

Fig. 4.29b The parameter $(C_{nr}/C_{DO})_W$ for subsonic speeds.

Ref: Pamadi

Fig A25 - I

SM (Margen estático) = 0.2



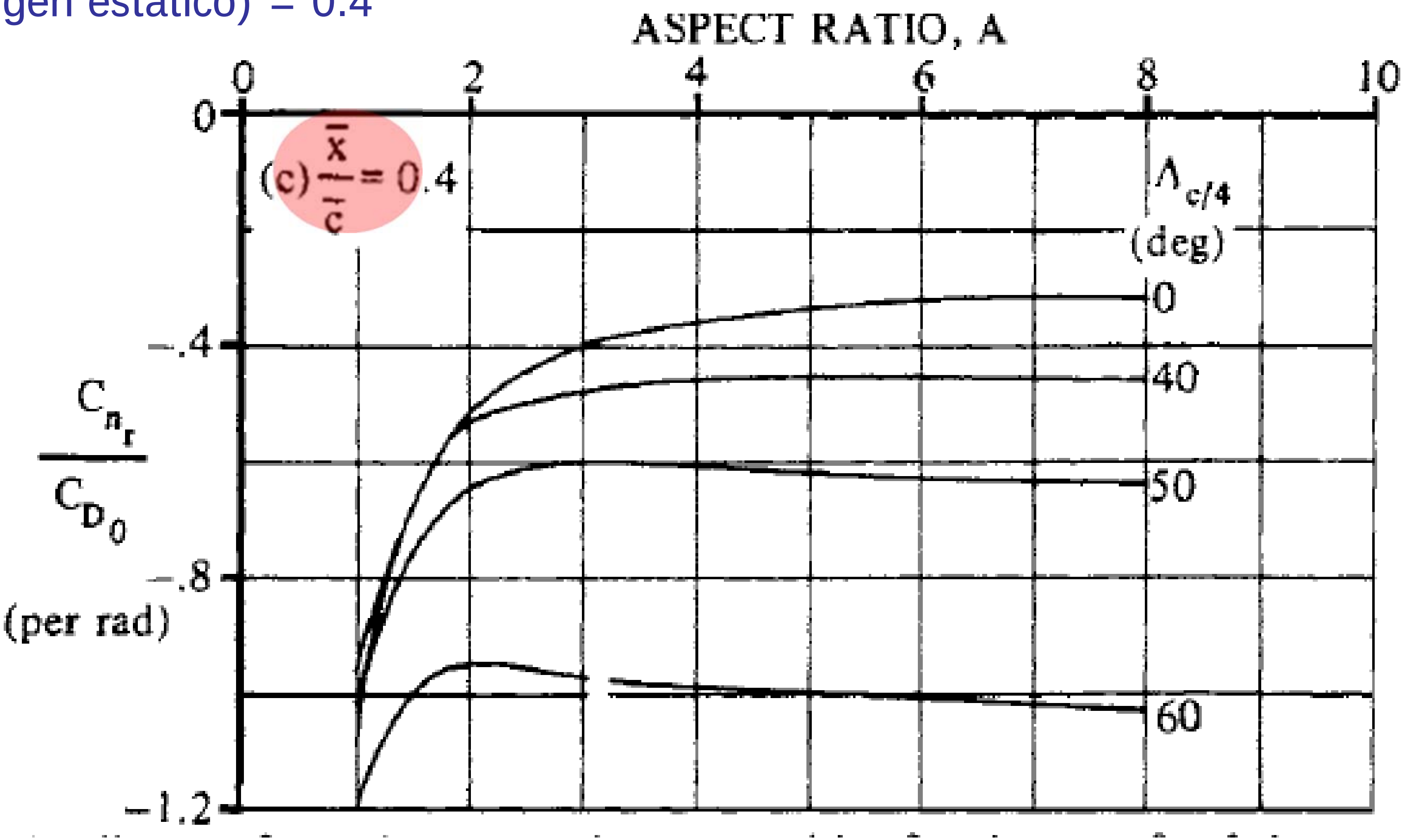
NOTE: $\bar{x}$ is the distance from the c.g. to the a.c., positive for the a.c. aft of the c.g.
$\bar{c}$ is the wing mean aerodynamic chord.

Fig. 4.29b The parameter $(C_{nr}/C_{DO})_W$ for subsonic speeds.

Ref: Pamadi

Fig A25 - I

SM (Margen estático) = 0.4



NOTE: $\bar{x}$ is the distance from the c.g. to the a.c., positive for the a.c. aft of the c.g.
$\bar{c}$ is the wing mean aerodynamic chord.

Fig. 4.29b The parameter $(C_{nr}/C_{DO})_W$ for subsonic speeds.

Ref: Pamadi

Contribución Vertical C_{N_r}

Tail contribution

Ref: Pamadi/Roskam

$$(C_{nr})_V = \frac{2}{b^2}(l_v \cos\alpha + z_v \sin\alpha)^2 C_{y\beta,V}$$ per radia

α ángulo de ataque de trimado asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline

$$l_v = X_{ac_v} - X_{cg} \qquad z_v = Z_{ac_v} - Z_{cg}$$

$$C_{y\beta,V} = -ka_v\left(1 + \frac{\partial\sigma}{\partial\beta}\right)\eta_v\frac{S_v}{S}$$

Derivadas $C_{Y_{\dot{\beta}}}, C_{L_{\dot{\beta}}}, C_{N_{\dot{\beta}}}$

Estimación Derivadas

- Contribución $C_{Y_{\dot{\beta}}}$
  - Vertical
- Contribución $C_{L_{\dot{\beta}}}$
  - Vertical
- Contribución $C_{N_{\dot{\beta}}}$
  - Vertical

Contribución Vertical $C_{Y_{\dot\beta}}$

Tail contribution

$$(C_{y\dot\beta})_V = 2a_v\sigma_\beta\left(\frac{S_v}{S}\right)\left[\frac{l_v\cos\alpha + z_v\sin\alpha}{b}\right]$$

α ángulo de ataque de trimado asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline $l_v = X_{ac_v} - X_{cg}$ $z_v = Z_{ac_v} - Z_{cg}$

$$\sigma_\beta = \sigma_{\beta\alpha}\alpha + \sigma_{\beta\Gamma}\Gamma + \sigma_{\beta,WB}$$

where α is the angle of attack in degrees and Γ is the dihedral angle in degrees.

$\sigma_{\beta\alpha}$ gives the variation of the sidewash with angle of attack, ⇒ Fig A26

$\sigma_{\beta\Gamma}$ represents the influence of dihedral angle Γ on the sidewash, ⇒ Fig A27

$\sigma_{\beta,WB}$ represents the influence of the wing–body interference effect on the sidewash. ⇒ Fig A28

Ref: Pamadi

Fig A26

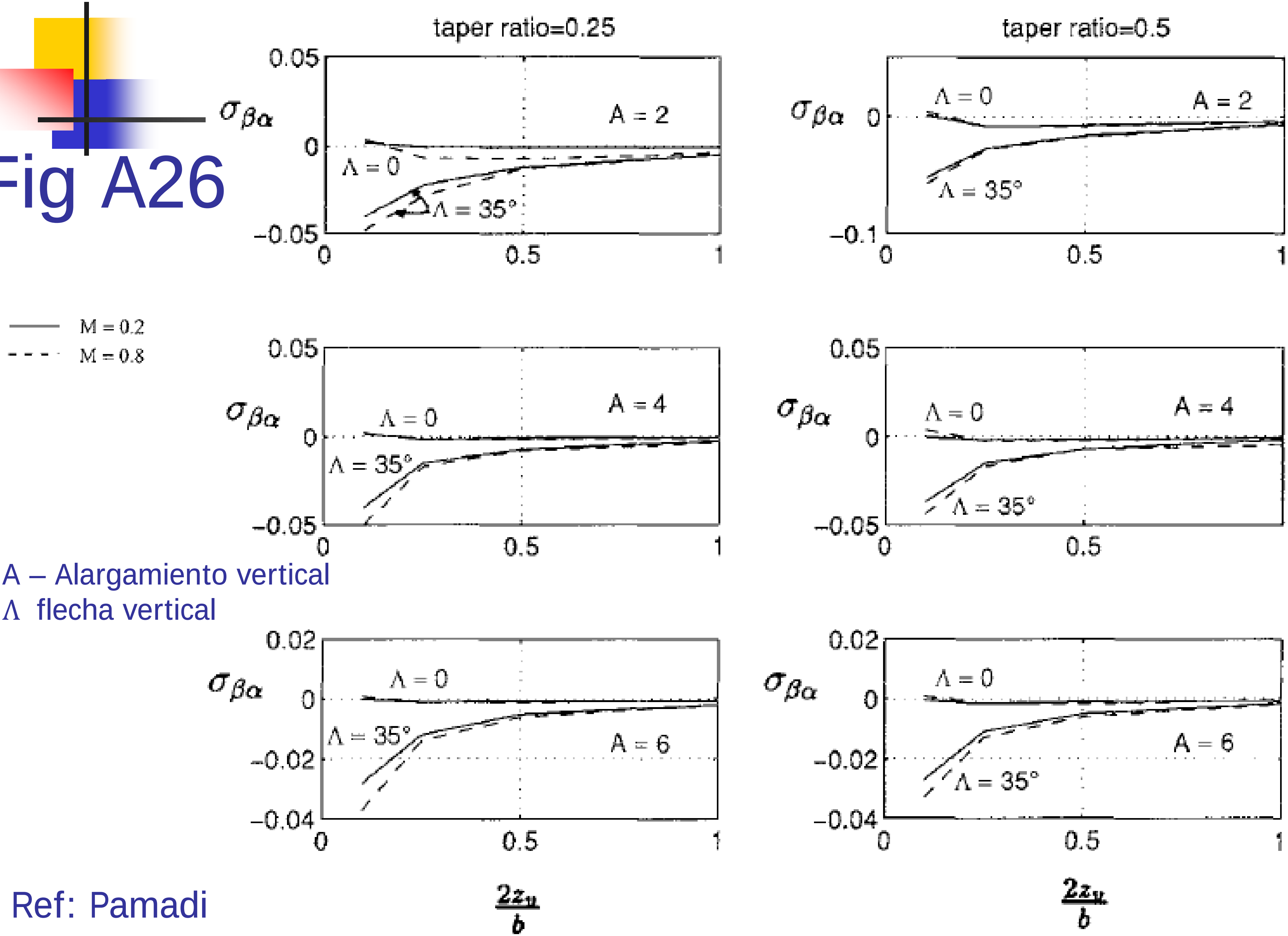


Fig. 4.30 The parameter $\sigma_{\beta\alpha}$ (per deg) at subsonic speeds.

A – Alargamiento vertical
Λ flecha vertical

Ref: Pamadi

Fig A27

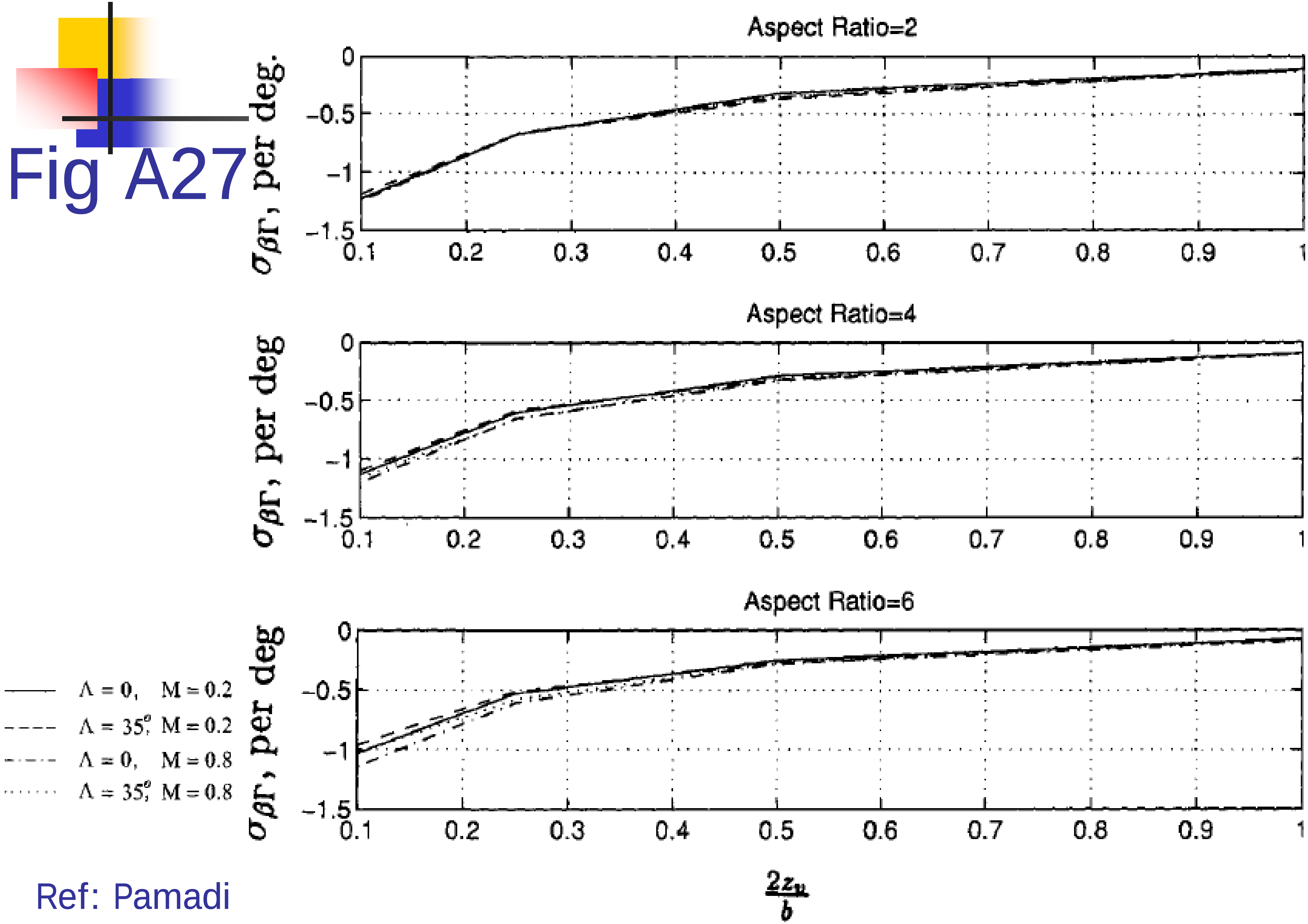


Fig. 4.31 The parameter $\sigma_{\beta\Gamma}$ at subsonic speeds.

Ref: Pamadi

Fig A28

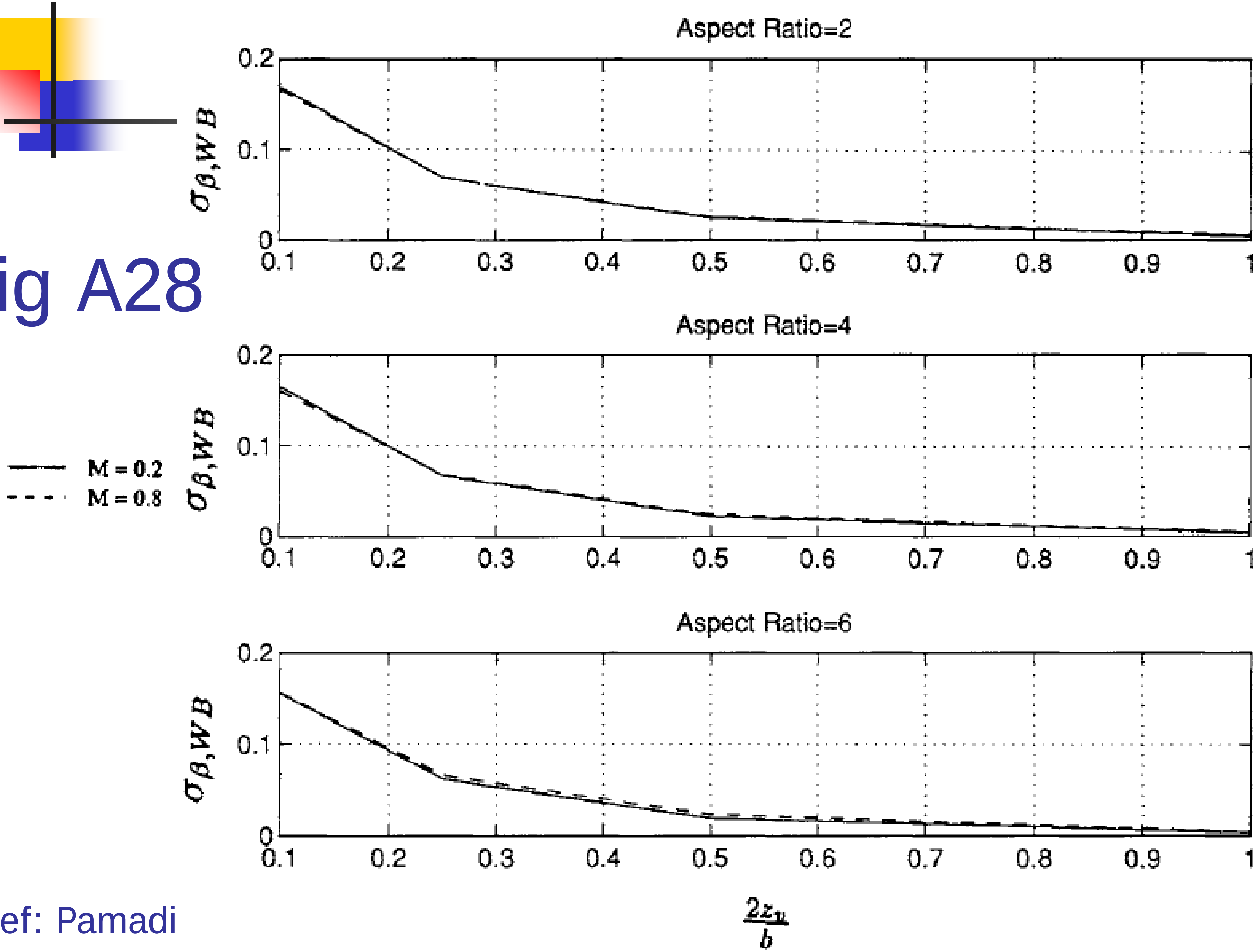


Fig. 4.32 The parameter $(\sigma_\beta)_{WB}$ at subsonic speeds.

Ref: Pamadi

Contribución Vertical $C_{L_{\dot{\beta}}} C_{N_{\dot{\beta}}}$

$$(C_{l\dot{\beta}})_V = (C_{y\dot{\beta}})_V \left[\frac{z_v \cos\alpha - l_v \sin\alpha}{b} \right]$$

$$(C_{n\dot{\beta}})_V = -(C_{y\dot{\beta}})_V \left[\frac{l_v \cos\alpha + z_v \sin\alpha}{b} \right]$$

$$(C_{y\dot{\beta}})_V = 2 a_v \sigma_\beta \left(\frac{S_v}{S} \right) \left[\frac{l_v \cos\alpha + z_v \sin\alpha}{b} \right]$$

α ángulo de ataque de trimado asumir que el estudio se realiza para $\alpha = 0$

z_v is the vertical distance between the aerodynamic center of the vertical tail and the center of gravity measured perpendicular to the fuselage centerline,
l_v is the corresponding horizontal distance measured parallel to the fuselage centerline

$$l_v = X_{ac_v} - X_{cg} \qquad z_v = Z_{ac_v} - Z_{cg}$$

Lateral-directional acceleration derivatives $C_{Y_{\dot\beta}}$

$$(C_{y\dot\beta})_V = 2a_v\sigma_\beta\left(\frac{S_v}{S}\right)\left[\frac{l_v\cos\alpha + z_v\sin\alpha}{b}\right]$$

a_v is the lift-curve slope of the vertical tail

$$\sigma_\beta = \sigma_{\beta\alpha}\alpha + \sigma_{\beta\Gamma}\left(\frac{\Gamma}{57.3}\right) + \sigma_{\beta,WB}$$

α is the angle of attack in degrees

Γ is the dihedral angle in degrees.

$\sigma_{\beta\alpha}$ gives the variation of the sidewash with angle of attack,

$\sigma_{\beta\Gamma}$ represents the influence of dihedral angle Γ on the sidewash,

$\sigma_{\beta,WB}$ represents the influence of the wing–body interference effect on the sidewash.

Fig A26

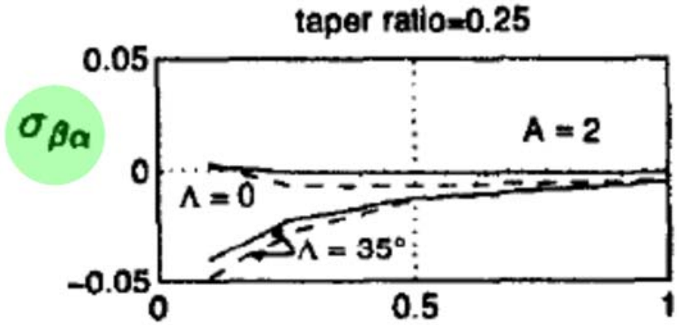


Fig A27

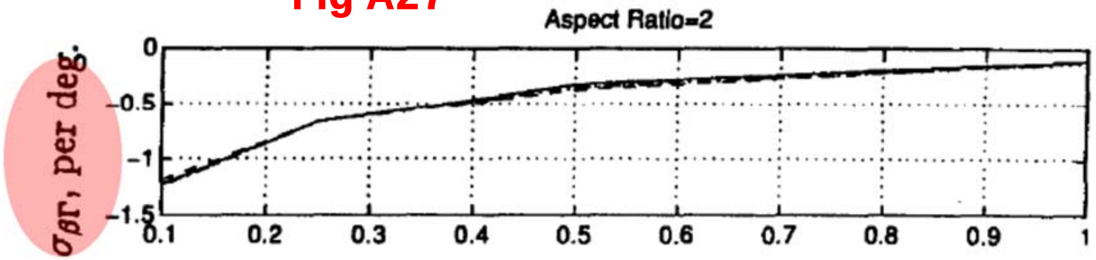
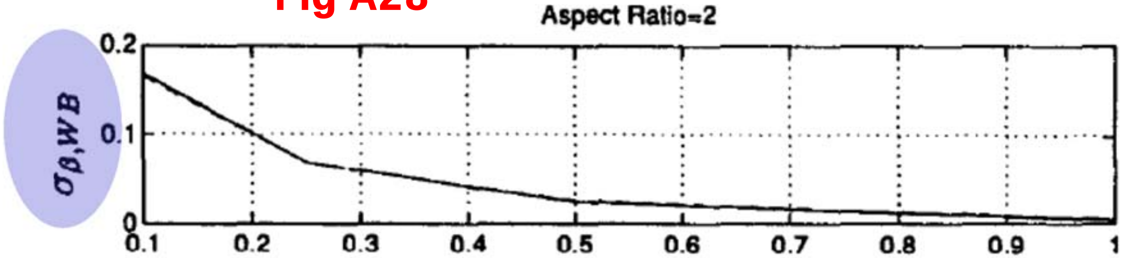


Fig A28



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